Laboratory #5

Issued: Sep. 14, 2009

Due: (Section A) Sep. 21, (B) Sep. 22, (C) Sep. 23, 2009

This lab uses MATLAB for basic signal processing.

Reading

Information about MATLAB commands can be obtained by typing `help COMMAND` at the prompt. For example:

```
>>help filter
```

Discrete-Time Signal Processing

Filtering

A DT LTI system can be described by a linear, constant coefficient difference equation (LCCDE)

$$y[n] + a_1 y[n-1] + a_2 y[n-2] + \cdots + a_L y[n-L] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]. \quad (1)$$

The output or response to the system is governed by the coefficients of the LCCDE. MATLAB uses the 'filter' command to compute the output of the LCCDE, for e.g.

```
>>y = filter(b,a,x);
```

where $\mathbf{x}$ is a vector of input samples, $\mathbf{b} = [b_0; b_1; \ldots; b_M]$ is a vector of feedforward coefficients, $\mathbf{a} = [1; a_1; a_2; \ldots; a_L]$ is a vector of feedback coefficients, and $\mathbf{y}$ is a vector of output samples returned by MATLAB. By convention we require all vectors to be column vectors.

Impulse Response

To compute the impulse response, we simply feed the system a unit-pulse sequence and measure what comes out. With MATLAB, you can numerically compute the impulse response of a LCCDE as follows. Consider a system described by the LCCDE

$$y[n] - \frac{1}{2} y[n-1] = x[n] \quad (2)$$

with initial, at rest conditions, $y[-1] = 0$. Following the above, we have would have in MATLAB

```
>>N = 100;
>>b = [1];
>>a = [1;-0.5];
>>delta = [1;zeros(N-1,1)];  % unit-pulse sequence is N samples long
>>h = filter(b,a,delta);
```

Note that for this system which is IIR, we can only compute a finite number of terms in the impulse response. We could also plot the impulse response if desired.

Step Response

Similarly, we could obtain the step response of the system by simply feeding the system a unit-step sequence and measure what comes out. With MATLAB, you can numerically compute the step response of a LCCDE as follows. For the same system as above, we have would have in MATLAB

```
>>N = 100;
>>b = [1];
>>a = [1;-0.5];
>>u = ones(N,1);  % unit-step sequence is N samples long
>>s = filter(b,a,u);
```

We could then plot the step response if desired.

Convolution

MATLAB provides a 'conv' command which computes the convolution between 2 signals

```
>>c = conv(a,b)
```

where **c** is the result of convolving **a** and **b**.

Continuous-Time Signal Processing

Filtering

A CT LTI system can be discribed by linear constant-coefficient differential equation (LCCDE) of the form

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}. \tag{3}$$

The output or response to the system is governed by the coefficients of the LCCDE. Due to the fact that MATLAB is executed on a digital computer, we can only approximate or simulate the behavior of CT LTI system. MATLAB uses the 'lsim' command to compute the output of the LCCDE for a given input

```
>>y = lsim(b,a,x,t)
```

where **x**, **y** is a vector of input, output samples which approximates the CT input, ouput signal respectively at specific points in time, $\mathbf{t} = [t_0,\ t_1,\ \ldots\ t_{N-1}]$; $\mathbf{b} = [b_0,\ b_1,\ \ldots\ b_M]$ is a row vector of feedforward coefficients; and $\mathbf{a} = [1,\ a_1,\ a_2,\ \ldots\ a_L]$ is a row vector of feedback coefficients.

Impulse Response

To compute the impulse response of a CT system, MATLAB uses the 'impulse' function

```
>>h = impulse(b,a,t)
```

Consider a system described by the LCCDE

$$\frac{dy(t)}{dt} + \frac{1}{2}y(t) \quad = \quad x(t). \tag{4}$$

The impulse response of this system can be computed in MATLAB with the commands

```
>>t = [0:10];
>>b = [1];
>>a = [0.5, 1];
>>h = impulse(b,a,t);
```

Step Response

To compute the step response of a CT system, MATLAB uses the 'step' function

```
>>s = step(b,a,t)
```

For the same system above, we would have in MATLAB

```
>>t = [0:10];
>>b = [1];
>>a = [0.5, 1];
>>s = step(b,a,t);
```

Notes

Pre-lab problems are due at the beginning of lab; regular problems are due at the end of lab but should be worked on before lab. If you need assistance with pre-lab problems, please contact teaching assistants or Prof. De Leon during office hours or by email.

Your solution to the lab should be a printout of the command you typed in or script and the answer or plot returned from MATLAB.

Pre-Lab Problems

1) For the following system

$$y[n] = -\frac{1}{3}y[n-2] + 0.5y[n-1] + \frac{1}{2}x[n] - \frac{1}{2}x[n-1] \quad (5)$$

use MATLAB 'filter' to determine and plot the (a) impulse response, (b) step response, and (c) output signal for the given input speech signal from Homework #3. Assume initial, at rest conditions.

2) For the following system

$$y[n] = 2x[n] - 1.5x[n-1] - 0.5x[n-2] \quad (6)$$

use MATLAB 'filter' to determine and plot the (a) impulse response, (b) step response, and (c) output signal for the given input speech signal from Homework #3. Assume initial, at rest conditions.

Regular Problems

1) For the following system

$$\frac{d^2y(t)}{dt^2} + \frac{1}{2}\frac{dy(t)}{dt} + \frac{1}{3}y(t) = \frac{1}{2}x(t) + \frac{dx(t)}{dt} \quad (7)$$

use MATLAB to determine and plot the (a) impulse response (plot for t=[0:0.01:10]) and (b) step response. Assume initial, at rest conditions.

2) For the following system

$$y[n] = \frac{1}{12} \sum_{k=n-11}^{n} x[k] \tag{8}$$

use MATLAB 'filter' to determine and plot the (a) impulse response, (b) step response, and (c) output signal for the given input speech signal from Homework #3. Assume initial, at rest conditions.

3) Using the impulse response computed in 2) determine the step response using the conv function

```
>>N = 150;
>>u = ones(N,1);  % unit-step sequence is N samples long
>>s = conv(h,u);
```

Compare your result to that in 2).

4) For the system in Lab, Prob 3), use MATLAB to determine and plot the output signal for

$$x(t) = e^{-t} \sin(2t) u(t) \tag{9}$$

for time points $\mathbf{t} = [0 : 0.1 : 10]$.