Laboratory #1

Issued: Aug. 22, 2009

Due: (Section A) Aug. 24, (B) Aug. 25, (C) Aug. 26, 2009

This lab will consist of a review of topics in mathematics necessary for this course.

Reading

Methods for the solution to these review problems can be found in undergraduate-level texts in calculus, complex variables, and linear algebra.

Notes

Pre-lab problems are due at the beginning of lab; regular problems are due at the end of lab but should be worked on before lab. If you need assistance with pre-lab problems, please contact teaching assistants or Dr. De Leon during office hours or by email.

Pre-Lab Problems

1. $\displaystyle\int_0^t \tau d\tau$

2. $\displaystyle\int_0^1 te^{-t/3}dt$

3. Show that if $|\alpha| < 1$, then $\displaystyle\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$.

4. Prove the validity of the following expression:

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N, & \alpha = 1 \\ \frac{1-\alpha^N}{1-\alpha}, & \text{for any complex number } \alpha \neq 1 \end{cases}$$

5. Determine the partial fraction coefficients A, B:
$$\frac{1}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

6. Determine the partial fraction coefficients A, B:
$$\frac{x+3}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

7. Determine the solution y to the following linear, constant-coefficient differential equation. Assume for your solution that when $x = 0$, $y = 1$.
$$\frac{dy}{dx} + 2y = e^x$$

8. Let $f(x) = \frac{1}{1+jx}$. Sketch $|f(x)|$ for $0 \le x < \infty$ [$|f(x)|$ denotes magnitude of $f(x)$]. Be sure to carefully label your plot and note the values of $|f(0)|$, $|f(1)|$, and $|f(\infty)|$.

9. Let $f(x) = \frac{1}{1+j100x}$. Sketch $|f(x)|$ for $0 \le x < \infty$. Be sure to carefully label your plot and note the values of $|f(0)|$, $|f(1)|$, and $|f(\infty)|$.

10. Let z, z_1, and z_2 be arbitrary complex numbers with the general form $z = x + jy = re^{j\theta}$ and let $z^* = x - jy = re^{-j\theta}$ represent the complex conjugate. Derive the following relations:

    $zz^* = r^2$, $z + z^* = 2\mathcal{R}e\{z\}$, $(z_1 + z_2)^* = z_1^* + z_2^*$, $|z| = |z^*|$

Regular Problems

Solve the following problems.

1. $\displaystyle\int_0^\infty e^{-t/2} dt$

2. Show that if $|\alpha| < 1$, then $\displaystyle\sum_{n=1}^{\infty} \alpha^n = \frac{\alpha}{1-\alpha}$.

3. Find

    $$\lim_{x \to \pi/2} \frac{\cos x}{\pi - 2x}$$

4. Find

    $$\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$$

5. Using Euler's identity, express in rectangular form the following ($j = \sqrt{-1}$)

    e^{j0}, $e^{j\pi/4}$, $e^{j\pi/2}$, $e^{j3\pi/4}$, $e^{j\pi}$, $e^{-j\pi}$, $e^{j2\pi}$

6. Determine the magnitude and phase angle of the following complex numbers

    1, $1+j$, j, $1-j$, -1, $-j$

7. Let

    $$x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

    Determine, x^T, $x + x$, $4x$, $x^T x$ where ^{T} denotes vector/matrix transpose.

8. Let

    $$A = \begin{bmatrix} 1 & 4 \\ 5 & -1 \end{bmatrix}$$

    Determine, A^T, $A + A$, $4A$, $A \cdot A$

9. Let z, z_1, and z_2 be arbitrary complex numbers with the general form $z = x + jy = re^{j\theta}$, let a be an arbitraty real number, and let $z^* = x - jy = re^{-j\theta}$ represent the complex conjugate. Derive the following relations:

   $z - z^* = 2j\mathcal{I}m\{z\}$, $(az_1z_2)^* = az_1^*z_2^*$, $(e^z)^* = e^{z^*}$, $|z_1z_2| = |z_1||z_2|$

