Homework 9 Solution

EE312 Fall 2015

Textbook Problem Solution:

4.21. (a) The given signal is

$$e^{-\alpha t}\cos(\omega_0 t)u(t) = \frac{1}{2}e^{-\alpha t}e^{j\omega_0 t}u(t) + \frac{1}{2}e^{-\alpha t}e^{-j\omega_0 t}u(t).$$

Therefore,

$$X(j\omega) = \frac{1}{2}\left[\frac{1}{\alpha - j(\omega_0 - \omega)} + \frac{1}{\alpha + j(\omega_0 + \omega)}\right]$$

4.33. (a) Taking the Fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 2j\omega + 8}.$$

Using partial fraction expansion, we obtain

$$H(j\omega) = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}.$$

Taking the inverse Fourier transform,

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t).$$

(b) For the given signal $x(t)$, we have

$$X(j\omega) = \frac{1}{(2 + j\omega)^2}.$$

Therefore,

$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{2}{(-\omega^2 + 2j\omega + 8)}\frac{1}{(2 + j\omega)^2}.$$

Using partial fraction expansion, we obtain

$$Y(j\omega) = \frac{1/4}{j\omega + 2} - \frac{1/2}{(j\omega + 2)^2} + \frac{1}{(j\omega + 2)^3} - \frac{1/4}{j\omega + 4}.$$

Taking the inverse Fourier transform,

$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t).$$

4.34. (a) We have

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.$$

Cross-multiplying and taking the inverse Fourier transform, we obtain

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t).$$

(b) We have

$$H(j\omega) = \frac{2}{2 + j\omega} - \frac{1}{3 + j\omega}.$$

Taking the inverse Fourier transform we obtain,

$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t).$$

(c) We have

$$X(j\omega) = \frac{1}{4 + j\omega} - \frac{1}{(4 + j\omega)^2}.$$

Therefore,

$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{(4 + j\omega)(2 + j\omega)}.$$

Finding the partial fraction expansion of $Y(j\omega)$ and taking the inverse Fourier transform,

$$y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t).$$

5.1. (a) Let $x[n] = (1/2)^{n-1}u[n-1]$. Using the Fourier transform analysis equation (5.9), the Fourier transform $X(e^{j\omega})$ of this signal is

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= \sum_{n=1}^{\infty}(1/2)^{n-1}e^{-j\omega n} \\ &= \sum_{n=0}^{\infty}(1/2)^{n}e^{-j\omega(n+1)} \\ &= e^{-j\omega}\frac{1}{(1-(1/2)e^{-j\omega})} \end{aligned}$$

5.2. (a) Let $x[n] = \delta[n-1] + \delta[n+1]$. Using the Fourier transform analysis equation (5.9), the Fourier transform $X(e^{j\omega})$ of this signal is

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= e^{-j\omega} + e^{j\omega} = 2\cos\omega \end{aligned}$$

5.3. We note from Section 5.2 that a periodic signal $x[n]$ with Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$

has a Fourier transform

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right).$$

(a) Consider the signal $x_1[n] = \sin(\frac{\pi}{3}n + \frac{\pi}{4})$. We note that the fundamental period of the signal $x_1[n]$ is $N = 6$. The signal may be written as

$$x_1[n] = (1/2j)e^{j(\frac{\pi}{3}n+\frac{\pi}{4})} - (1/2j)e^{-j(\frac{\pi}{3}n+\frac{\pi}{4})} = (1/2j)e^{j\frac{\pi}{4}}e^{j\frac{2\pi}{6}n} - (1/2j)e^{-j\frac{\pi}{4}}e^{-j\frac{2\pi}{6}n}.$$

From this, we obtain the non-zero Fourier series coefficients a_k of $x_1[n]$ in the range $-2 \le k \le 3$ as

$$a_1 = (1/2j)e^{j\frac{\pi}{4}}, \quad a_{-1} = -(1/2j)e^{-j\frac{\pi}{4}}.$$

Therefore, in the range $-\pi \le \omega \le \pi$, we obtain

$$\begin{aligned} X(e^{j\omega}) &= 2\pi a_1 \delta(\omega - \frac{2\pi}{6}) + 2\pi a_{-1}\delta(\omega + \frac{2\pi}{6}) \\ &= (\pi/j)\{e^{j\pi/4}\delta(\omega - 2\pi/6) - e^{-j\pi/4}\delta(\omega + 2\pi/6)\} \end{aligned}$$

5.21. (a) The given signal is

$$x[n] = u[n-2] - u[n-6] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5].$$

Using the Fourier transform analysis eq. (5.9), we obtain

$$X(e^{j\omega}) = e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega} + e^{-5j\omega}.$$

(f) The given signal is

$$x[n] = -3\delta[n+3] - 2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3].$$

Using the Fourier transform analysis eq. (5.9), we obtain

$$X(e^{j\omega}) = -3e^{3j\omega} - 2e^{2j\omega} - e^{j\omega} + e^{-j\omega} + 2e^{-2j\omega} + 3e^{-3j\omega}.$$

(h) The given signal is

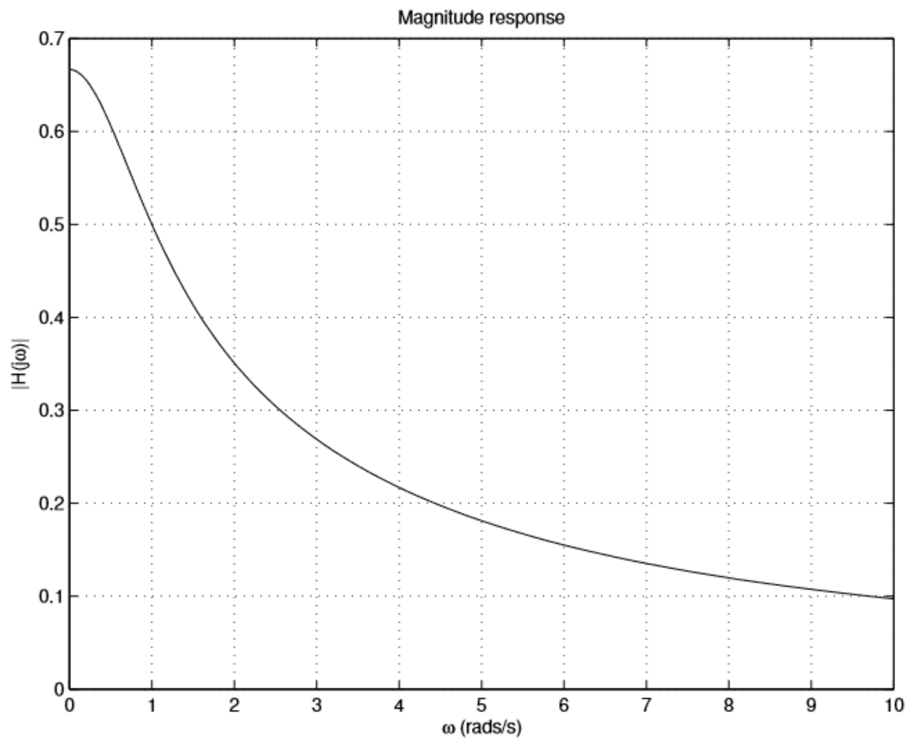
$$\begin{aligned} x[n] &= \sin(5\pi n/3) + \cos(7\pi n/3) \\ &= -\sin(\pi n/3) + \cos(\pi n/3) \\ &= -\frac{1}{2j}[e^{j\pi n/3} - e^{-j\pi n/3}] + \frac{1}{2}[e^{j\pi n/3} + e^{-j\pi n/3}]. \end{aligned}$$

Therefore,

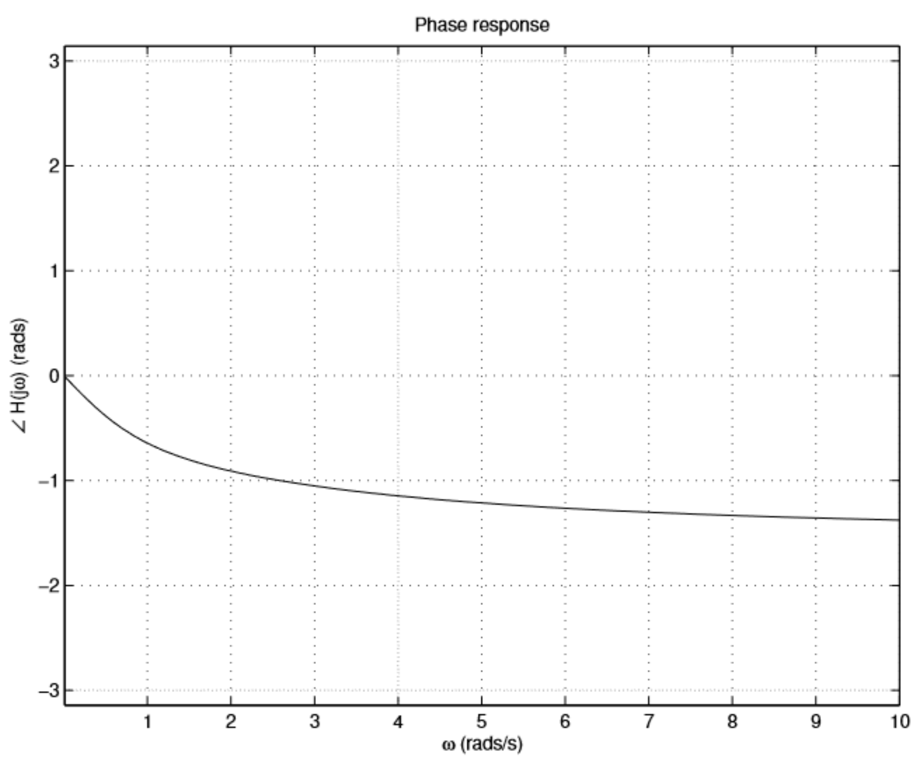
$$X(e^{j\omega}) = -\frac{\pi}{j}[\delta(\omega - \pi/3) - \delta(\omega + \pi/3)] + \pi[\delta(\omega - \pi/3) + \delta(\omega + \pi/3)], \quad \text{in } 0 \le |\omega| < \pi.$$

Software Problem Solution:

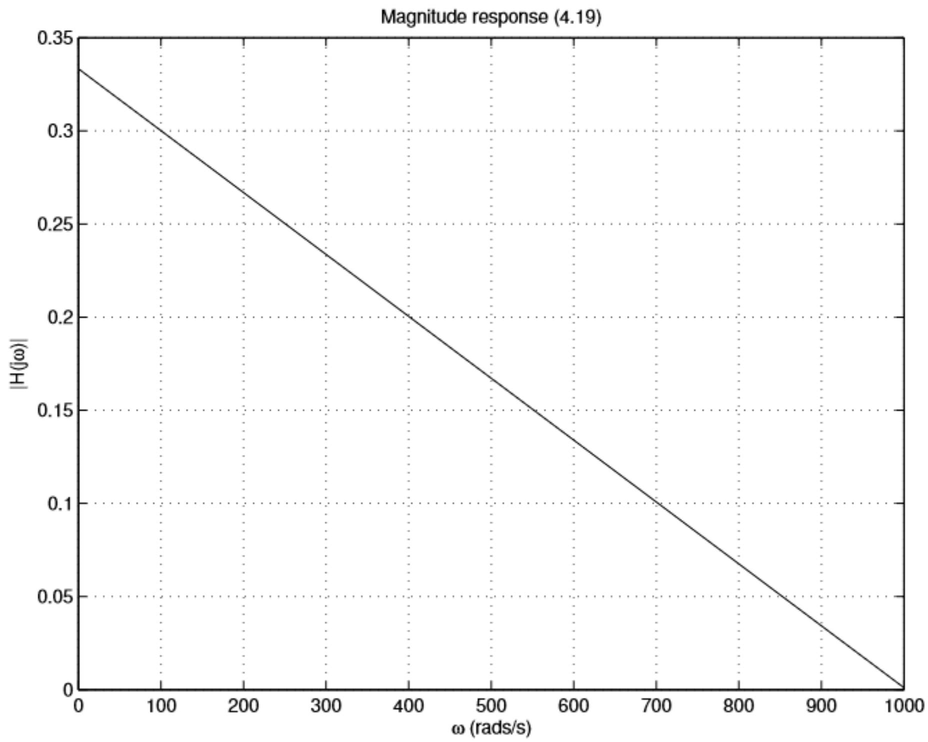
1 (a):



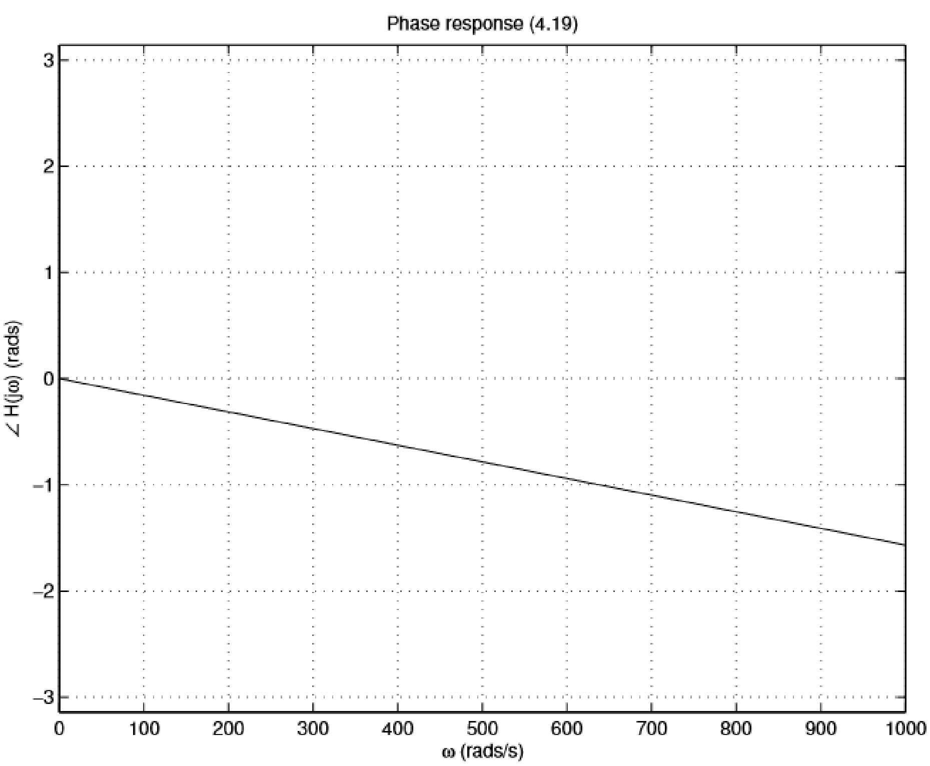
1 (b):



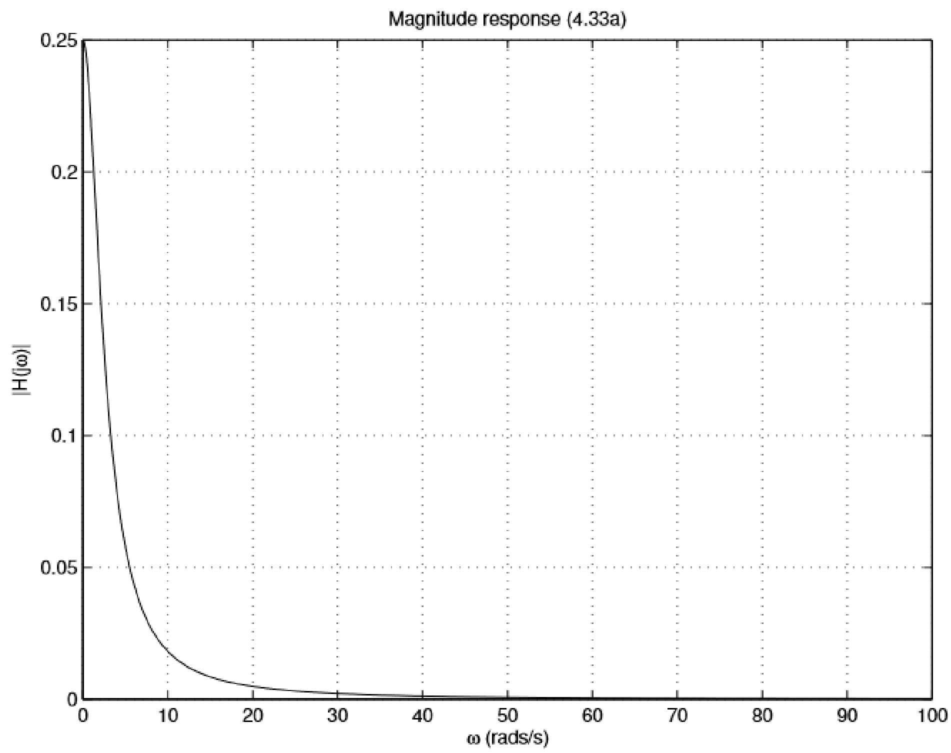
2 (a):


Magnitude response (4.19)
|H(jω)|
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 100 200 300 400 500 600 700 800 900 1000
ω (rads/s)

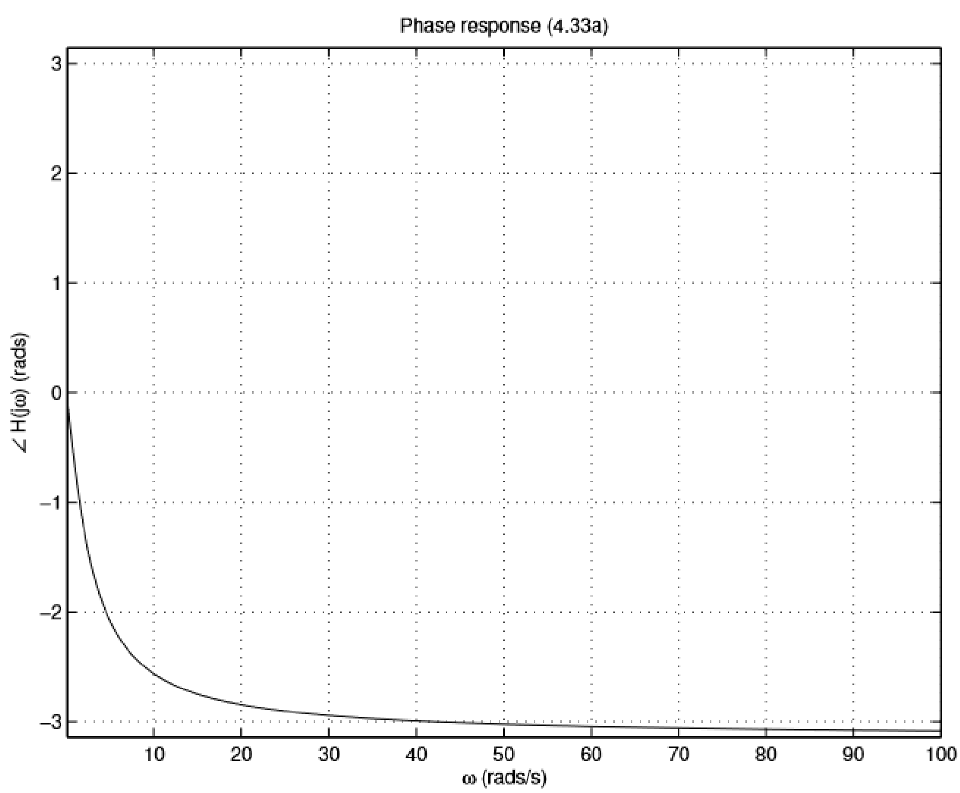
2 (b):


Phase response (4.19)
∠ H(jω) (rads)
3
2
1
0
−1
−2
−3
0 100 200 300 400 500 600 700 800 900 1000
ω (rads/s)

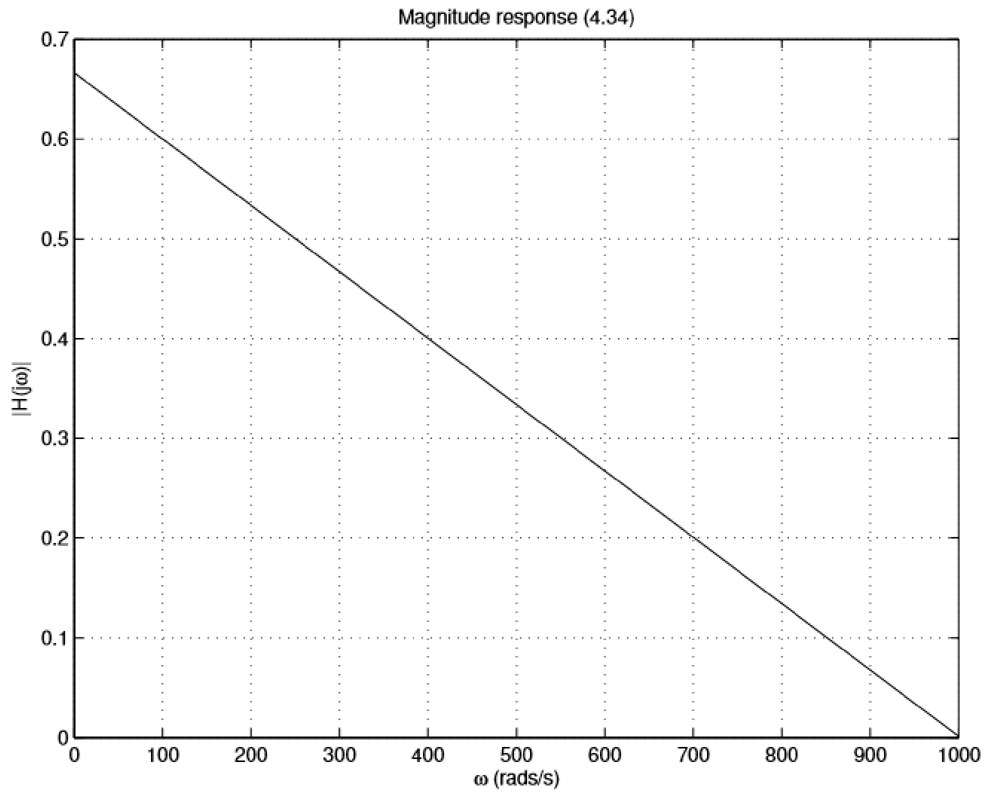
3 (a):


Magnitude response (4.33a)
|H(jω)|
ω (rads/s)
0.25
0.2
0.15
0.1
0.05
0
0 10 20 30 40 50 60 70 80 90 100

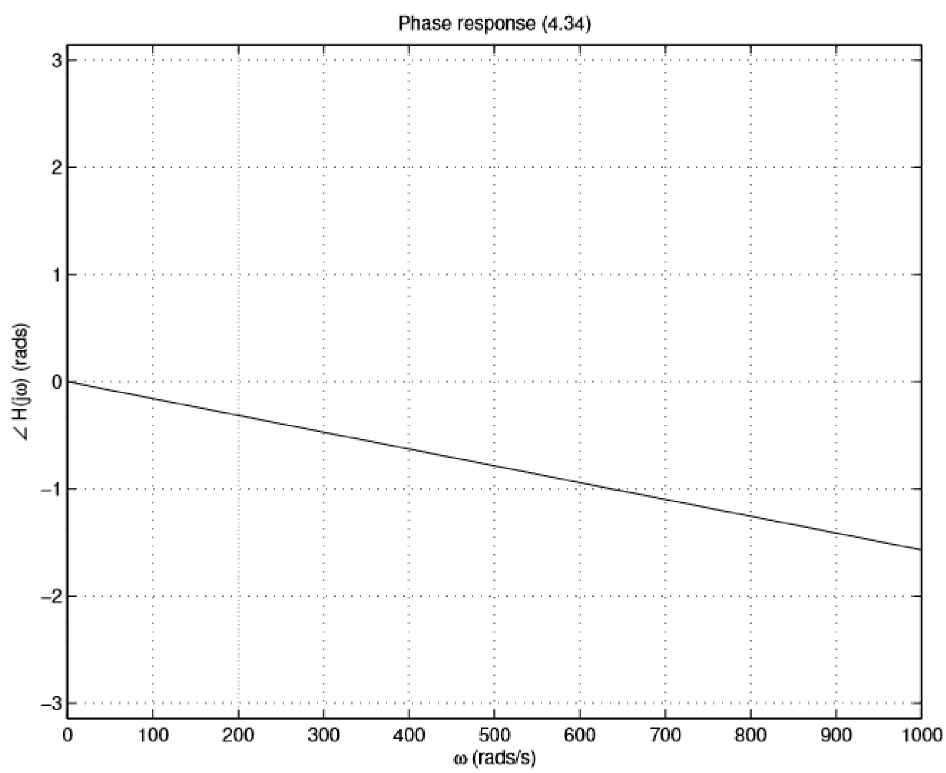
3 (b):


Phase response (4.33a)
∠ H(jω) (rads)
ω (rads/s)
3
2
1
0
−1
−2
−3
10 20 30 40 50 60 70 80 90 100

4 (a):


Magnitude response (4.34)
|H(jω)|
ω (rads/s)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
700
800
900
1000

4 (b):


Phase response (4.34)
∠ H(jω) (rads)
ω (rads/s)
3
2
1
0
−1
−2
−3
0
100
200
300
400
500
600
700
800
900
1000