Homework 8 solution

3.36. We will first evaluate the frequency response of the system. Consider an input $x[n]$ of the form $e^{j\omega n}$. From the discussion in Section 3.9 we know that the response to this input will be $y[n] = H(e^{j\omega})e^{j\omega n}$. Therefore, substituing these in the given difference equation, we get

$$H(e^{j\omega})e^{j\omega n} - \frac{1}{4}e^{-j\omega}e^{j\omega n}H(e^{j\omega}) = e^{j\omega n}.$$

Therefore,

$$H(j\omega) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}.$$

From eq. (3.131), we know that

$$y[n] = \sum_{k=<N>} a_k H(e^{j2\pi k/N})e^{jk(2\pi/N)n}$$

when the input is $x[n]$. $x[n]$ has the Fourier series coefficients a_k and fundamental frequency $2\pi/N$. Therefore, the Fourier series coefficients of $y[n]$ are $a_k H(e^{j2\pi k/N})$.

(a) Here, $N = 4$ and the nonzero FS coefficients of $x[n]$ are $a_3 = a^*_{-3} = 1/2j$. Therefore, the nonzero FS coefficients of $y[n]$ are

$$b_3 = a_1 H(e^{3j\pi/4}) = \frac{1}{2j(1-(1/4)e^{-j3\pi/4})}, \qquad b_{-3} = a_{-1}H(e^{-3j\pi/4}) = \frac{-1}{2j(1-(1/4)e^{j3\pi/4})}.$$

4.1. (a) Let $x(t) = e^{-2(t-1)}u(t-1)$. Then the Fourier transform $X(j\omega)$ of $x(t)$ is:

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} e^{-2(t-1)}u(t-1)e^{-j\omega t}dt \\ &= \int_{1}^{\infty} e^{-2(t-1)}e^{-j\omega t}dt \\ &= e^{-j\omega}/(2+j\omega) \end{aligned}$$

$|X(j\omega)|$ is as shown in Figure S4.1.

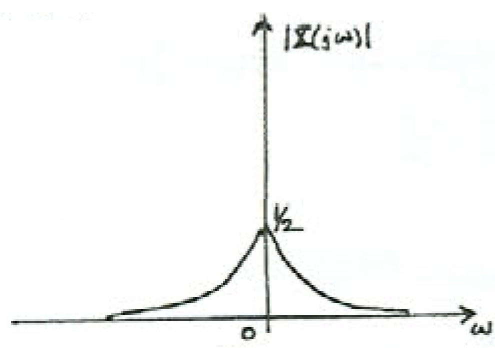


Figure S4.1

4.2. (a) Let $x_1(t) = \delta(t+1) + \delta(t-1)$. Then the Fourier transform $X_1(j\omega)$ of $x(t)$ is:

$$\begin{aligned} X_1(j\omega) &= \int_{-\infty}^{\infty} [\delta(t+1) + \delta(t-1)]e^{-j\omega t}dt \\ &= e^{j\omega} + e^{-j\omega} = 2\cos\omega \end{aligned}$$

$|X_1(j\omega)|$ is as sketched in Figure S4.2.

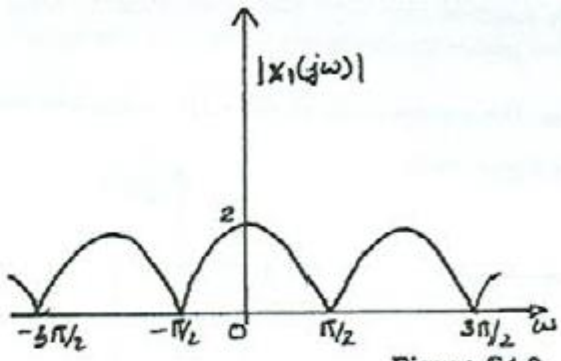


Figure S4.2

4.3. (a) The signal $x_1(t) = \sin(2\pi t + \pi/4)$ is periodic with a fundamental period of $T = 1$. This translates to a fundamental frequency of $\omega_0 = 2\pi$. The nonzero Fourier series coefficients of this signal may be found by writing it in the form

$$\begin{aligned} x_1(t) &= \frac{1}{2j}\left(e^{j(2\pi t+\pi/4)} - e^{-j(2\pi t+\pi/4)}\right) \\ &= \frac{1}{2j}e^{j\pi/4}e^{j2\pi t} - \frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t} \end{aligned}$$

Therefore, the nonzero Fourier series coefficients of $x_1(t)$ are

$$a_1 = \frac{1}{2j}e^{j\pi/4}e^{j2\pi t}, \qquad a_{-1} = -\frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t}$$

From Section 4.2, we know that for periodic signals, the Fourier transform consists of a train of impulses occurring at $k\omega_0$. Furthermore, the area under each impulse is 2π times the Fourier series coefficient a_k. Therefore, for $x_1(t)$, the corresponding Fourier transform $X_1(j\omega)$ is given by

$$\begin{aligned} X_1(j\omega) &= 2\pi a_1\delta(\omega - \omega_0) + 2\pi a_{-1}\delta(\omega + \omega_0) \\ &= (\pi/j)e^{j\pi/4}\delta(\omega - 2\pi) - (\pi/j)e^{-j\pi/4}\delta(\omega + 2\pi) \end{aligned}$$

4.4. (a) The inverse Fourier transform is

$$\begin{aligned} x_1(t) &= (1/2\pi)\int_{-\infty}^{\infty}[2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)]e^{j\omega t}d\omega \\ &= (1/2\pi)[2\pi e^{j0t} + \pi e^{j4\pi t} + \pi e^{-j4\pi t}] \\ &= 1 + (1/2)e^{j4\pi t} + (1/2)e^{-j4\pi t} = 1 + \cos(4\pi t) \end{aligned}$$

4.19. We know that

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}.$$

Since it is given that $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$, we can compute $Y(j\omega)$ to be

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}.$$

Since. $H(j\omega) = 1/(3 + j\omega)$, we have

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = 1/(4 + j\omega)$$

Taking the inverse Fourier transform of $X(j\omega)$, we have

$$x(t) = e^{-4t}u(t).$$

4.22)

(d) $x(t) = \frac{2j}{\pi} \sin t + \frac{3}{\pi} \cos(2\pi t)$

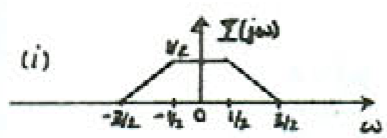
4.26. (a) (i) We have

$$\begin{aligned} Y(j\omega) &= X(j\omega)H(j\omega) = \left[\frac{1}{(2 + j\omega)^2}\right]\left[\frac{1}{4 + j\omega}\right] \\ &= \frac{(1/4)}{4 + j\omega} - \frac{(1/4)}{2 + j\omega} + \frac{(1/2)}{(2 + j\omega)^2} \end{aligned}$$

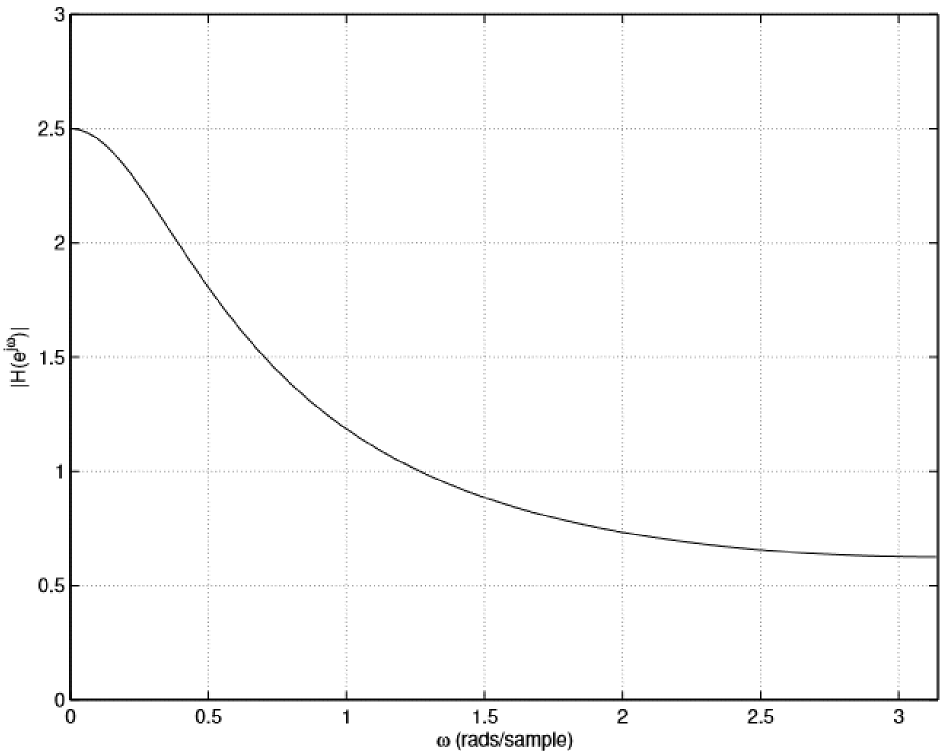
Taking the inverse Fourier transform we obtain

$$y(t) = \frac{1}{4}e^{-4t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t).$$

4.28(b) (i)

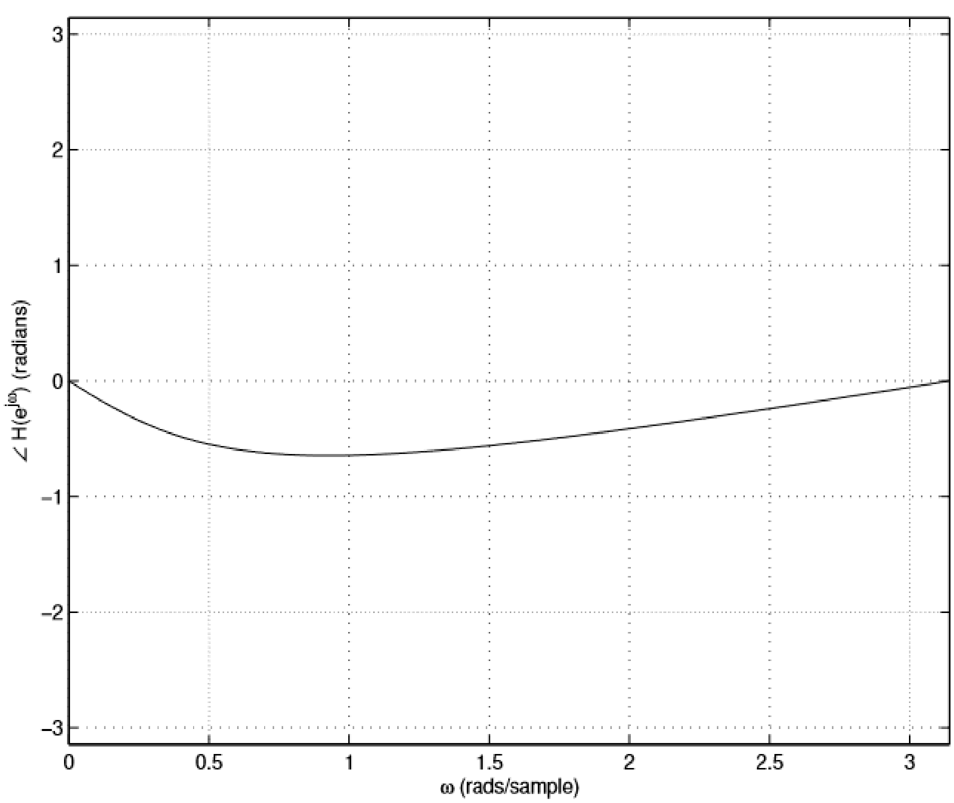


1(a):



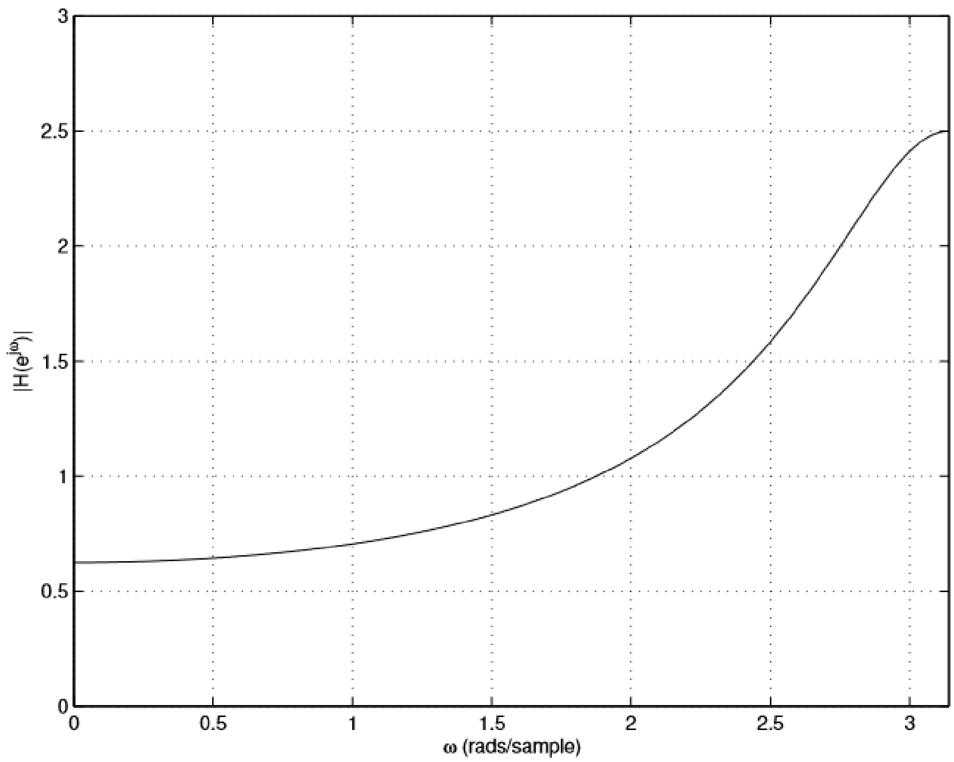
3
2.5
2
1.5
1
0.5
0
$|H(e^{j\omega})|$
0
0.5
1
1.5
2
2.5
3
ω (rads/sample)

1(b):



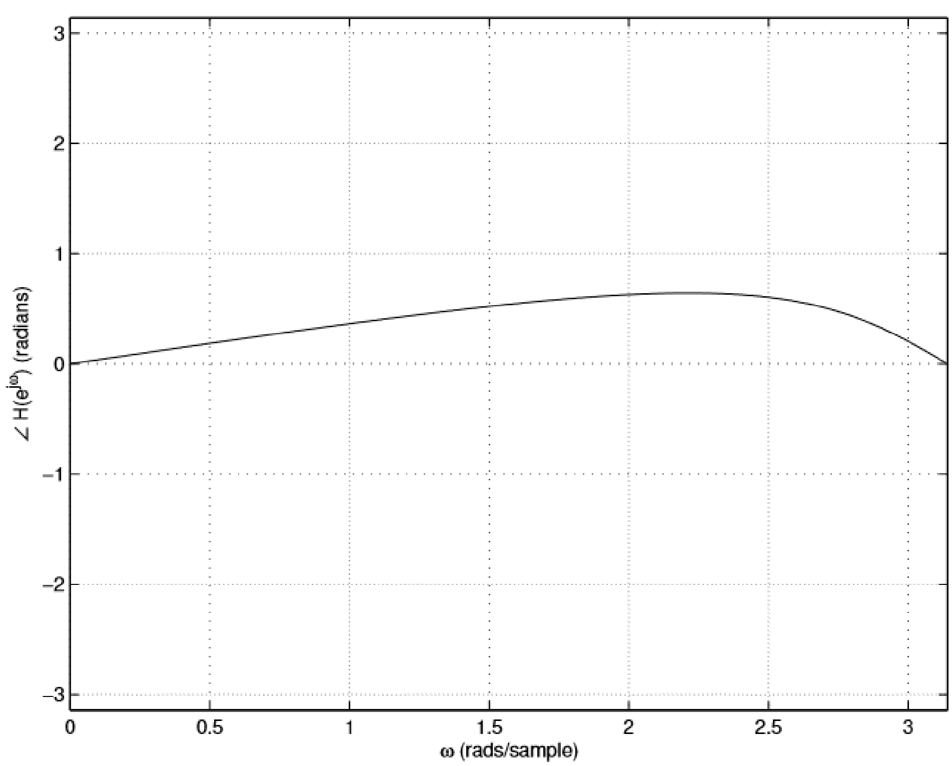
3
2
1
0
−1
−2
−3
$\angle H(e^{j\omega})$ (radians)
0
0.5
1
1.5
2
2.5
3
ω (rads/sample)

1(c):



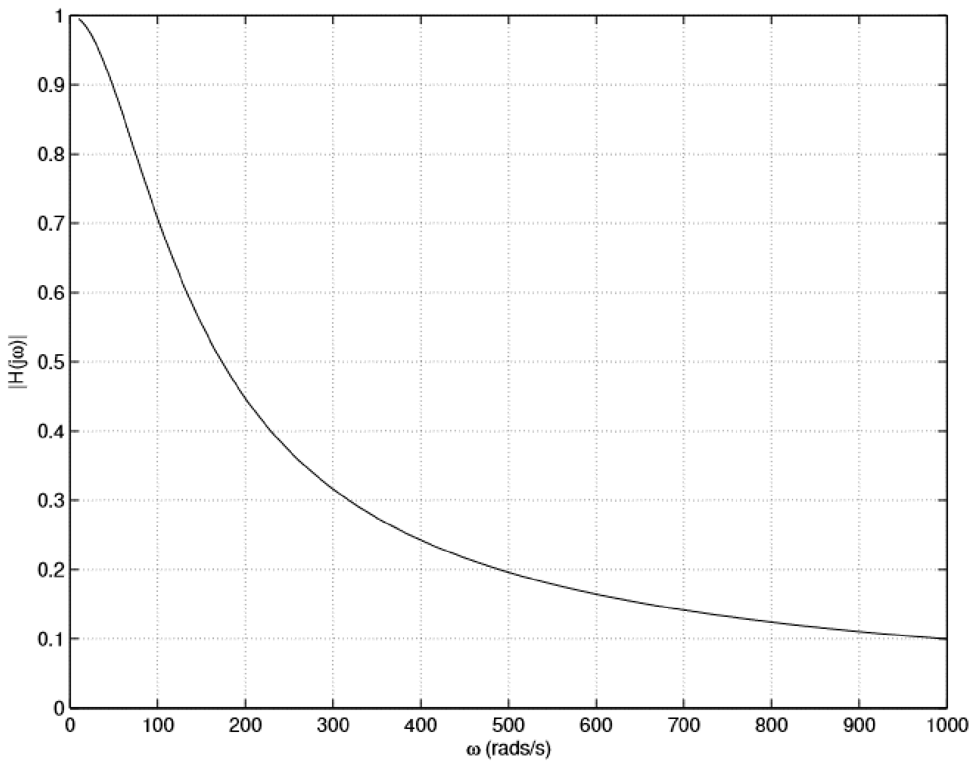
3
2.5
2
1.5
1
0.5
0
|H(e^jω)|
0
0.5
1
1.5
2
2.5
3
ω (rads/sample)

1(d):

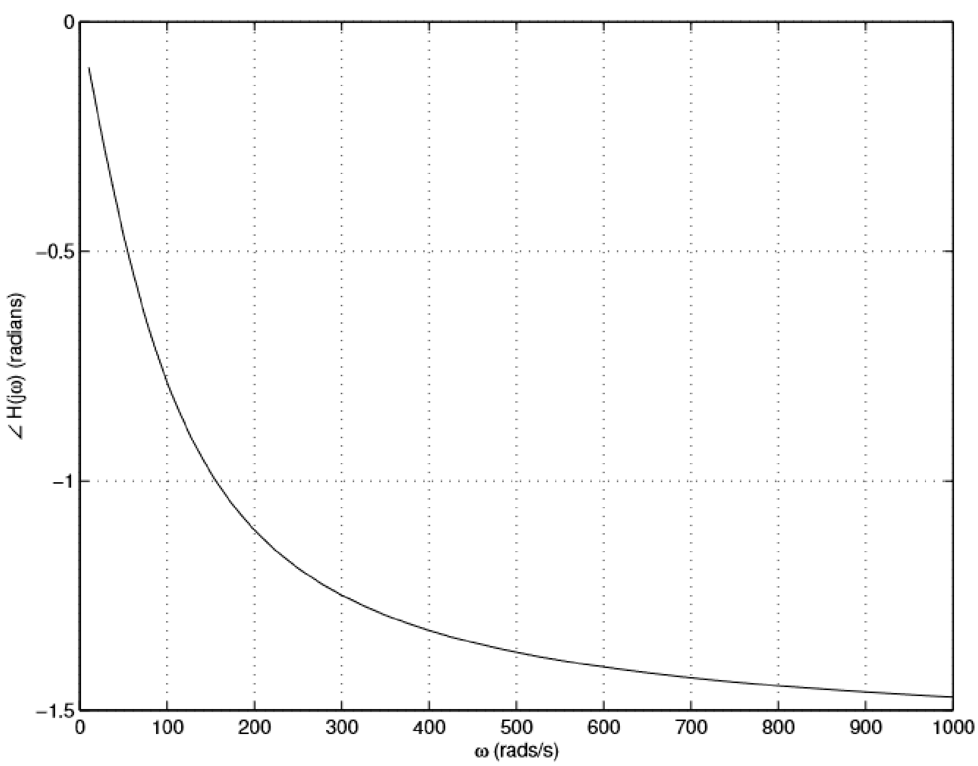


3
2
1
0
−1
−2
−3
∠ H(e^jω) (radians)
0
0.5
1
1.5
2
2.5
3
ω (rads/sample)

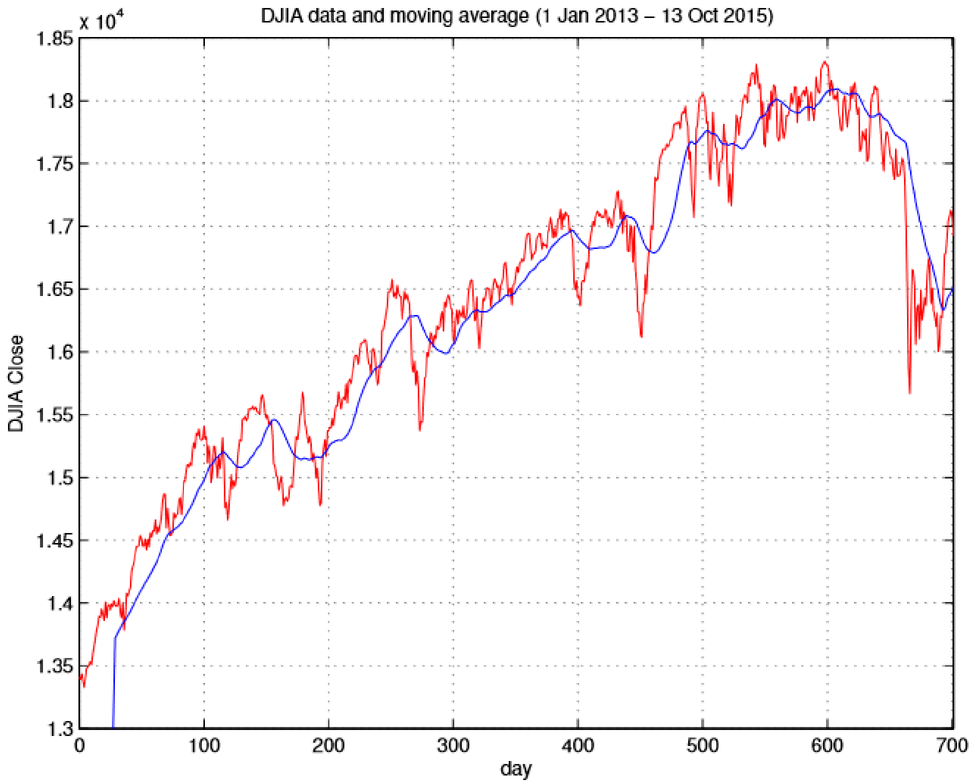
2(a):


|H(jω)|
ω (rads/s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
100
200
300
400
500
600
700
800
900
1000

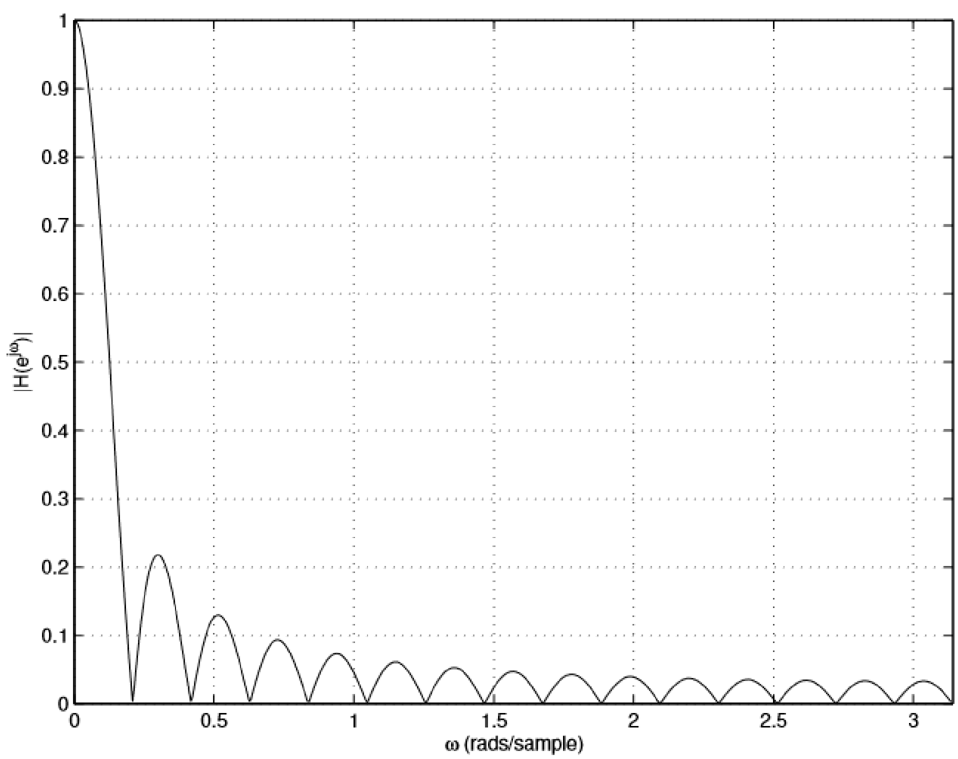
2(b):


∠ H(jω) (radians)
ω (rads/s)
0
−0.5
−1
−1.5
0
100
200
300
400
500
600
700
800
900
1000

3):


DJIA data and moving average (1 Jan 2013 – 13 Oct 2015)
x 10^4
1.85
1.8
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.3
DJIA Close
0
100
200
300
400
500
600
700
day

4):


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
|H(e^jω)|
0
0.5
1
1.5
2
2.5
3
ω (rads/sample)