Homework 7 Solution

EE312 Fall 215

3.10. Since the Fourier series coeffiecients repeat every N, we have

$$a_1 = a_{15}, \quad a_2 = a_{16} \quad, \text{and} \quad a_3 = a_{17}$$

Furthermore, since the signal is real and odd, the Fourier series coefficients a_k will be purely imaginary and odd. Therefore, $a_0 = 0$ and

$$a_1 = -a_{-1}, \quad a_2 = -a_{-2} \quad a_3 = -a_{-3}$$

Finally,

$$a_{-1} = -j, \quad a_{-2} = -2j, \quad a_{-3} = -3j$$

3.14. The signal $x[n]$ is periodic with period $N = 4$. Its Fourier series coefficients are

$$\begin{aligned} a_k &= \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}kn} \\ &= \frac{1}{4}, \quad \text{for all } k \end{aligned}$$

From the results presented in Section 3.8, we know that the output $y[n]$ is given by

$$\begin{aligned} y[n] &= \sum_{k=0}^{3} a_k H(e^{j(2\pi/4)k})e^{jk(2\pi/4)n} \\ &= \tfrac{1}{4}H(e^{j0})e^{j0} + \tfrac{1}{4}H(e^{j(\pi/2)})e^{j(\pi/2)} \\ &\quad + \tfrac{1}{4}H(e^{j(3\pi/2)})e^{j(3\pi/2)} + \tfrac{1}{4}H(e^{j(\pi)})e^{j(\pi)} \end{aligned} \qquad \text{(S3.14-1)}$$

From the given information, we know that $y[n]$ is

$$\begin{aligned} y[n] &= \cos(\frac{5\pi}{2}n + \frac{\pi}{4}) \\ &= \cos(\frac{\pi}{2}n + \frac{\pi}{4}) \\ &= \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})} \\ &= \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{j(3\frac{\pi}{2}n-\frac{\pi}{4})} \end{aligned}$$

Comparing this with eq. (S3.14-1), we have

$$H(e^{j0}) = H(e^{j\pi}) = 0$$

and

$$H(e^{j\frac{\pi}{2}}) = 2e^{j\frac{\pi}{4}}, \quad \text{and} \quad H(e^{3j\frac{\pi}{2}}) = 2e^{-j\frac{\pi}{4}}$$

3.16

(b) The signal $x_2[n]$ is periodic with period $N = 16$. The signal $x_2[n]$ may be written as

$$\begin{aligned} x_2[n] &= e^{j(2\pi/16)(0)n} - (j/2)e^{j(\pi/4)}e^{j(2\pi/16)(3)n} + (j/2)e^{-j(\pi/4)}e^{-j(2\pi/16)(3)n} \\ &= e^{j(2\pi/16)(0)n} - (j/2)e^{j(\pi/4)}e^{j(2\pi/16)(3)n} + (j/2)e^{-j(\pi/4)}e^{j(2\pi/16)(13)n} \end{aligned}$$

Therefore, the non-zero Fourier series coefficients of $x_2[n]$ in the range $0 \leq k \leq 15$ are

$$a_0 = 1, \quad a_3 = -(j/2)e^{j(\pi/4)}, \quad a_{13} = (j/2)e^{-j(\pi/4)}$$

Using the results derived in Section 3.8, the output $y_2[n]$ is given by

$$\begin{aligned} y_2[n] &= \sum_{k=0}^{15} a_k H(e^{j2\pi k/16}) e^{k(2\pi/16)} \\ &= 0 - (j/2)e^{j(\pi/4)} e^{j(2\pi/16)(3)n} + (j/2)e^{-j(\pi/4)} e^{j(2\pi/16)(13)n} \\ &= \sin(\frac{3\pi}{8}n + \frac{\pi}{4}) \end{aligned}$$

3.21. Using the Fourier series synthesis eq. (3.38),

$$\begin{aligned} x(t) &= a_1 e^{j(2\pi/T)t} + a_{-1} e^{-j(2\pi/T)t} + a_5 e^{j5(2\pi/T)t} + a_{-5} e^{-j5(2\pi/T)t} \\ &= je^{j(2\pi/8)t} - je^{-j(2\pi/8)t} + 2e^{j5(2\pi/8)t} + 2e^{-j5(2\pi/8)t} \\ &= -2\sin(\frac{\pi}{4}t) + 4\cos(\frac{5\pi}{4}t) \\ &= -2\cos(\frac{\pi}{4}t - \pi/2) + 4\cos(\frac{5\pi}{4}t). \end{aligned}$$

3.28. (a) $N = 7$,

$$a_k = \frac{1}{7} \frac{e^{-j4\pi k/7} \sin(5\pi k/7)}{\sin(\pi k/7)}.$$

3.29. (a) $N = 8$. Over one period $(0 \leq n \leq 7)$,

$$x[n] = 4\delta[n-1] + 4\delta[n-7] + 4j\delta[n-3] - 4j\delta[n-5].$$

(c) $N = 8$. Over one period $(0 \leq n \leq 7)$,

$$x[n] = 1 + (-1)^n + 2\cos(\frac{\pi n}{4}) + 2\cos(\frac{3\pi n}{4}).$$

3.33. We will first evaluate the frequency response of the system. Consider an input $x(t)$ of the form $e^{j\omega t}$. From the discussion in Section 3.9.2 we know that the response to this input will be $y(t) = H(j\omega)e^{j\omega t}$. Therefore, substituing these in the given differential equation, we get

$$H(j\omega)j\omega e^{j\omega t} + 4e^{j\omega t} = e^{j\omega t}.$$

Therefore,

$$H(j\omega) = \frac{1}{j\omega + 4}.$$

From eq. (3.124), we know that

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

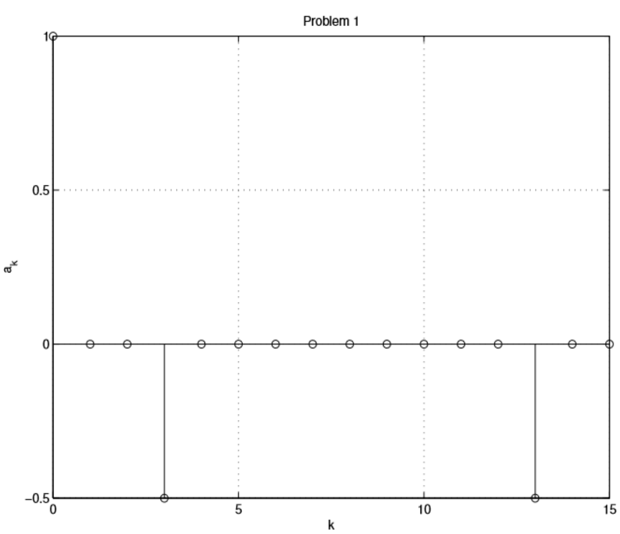
when the input is $x(t)$. $x(t)$ has the Fourier series coefficients a_k and fundamental frequency ω_0. Therefore, the Fourier series coefficients of $y(t)$ are $a_k H(jk\omega_0)$.

(a) Here, $\omega_0 = 2\pi$ and the nonzero FS coefficients of $x(t)$ are $a_1 = a_{-1} = 1/2$. Therefore, the nonzero FS coefficients of $y(t)$ are

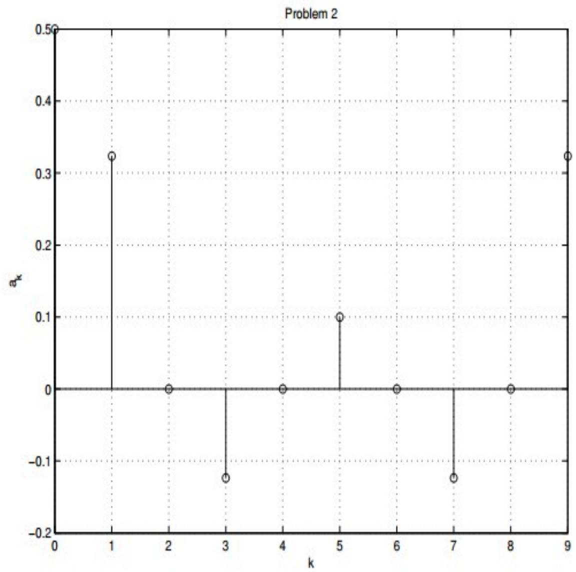
$$b_1 = a_1 H(j2\pi) = \frac{1}{2(4 + j2\pi)}, \qquad b_{-1} = a_{-1} H(-j2\pi) = \frac{1}{2(4 - j2\pi)}.$$

Software Problem Solution:

Problem 1.



Problem 2.



Problem 3