Fall 2015 – EE312

Homework #6 Solution

3.3. The given signal is

$$\begin{aligned} x(t) &= 2 + \frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t} - 2je^{j(5\pi/3)t} + 2je^{-j(5\pi/3)t} \\ &= 2 + \frac{1}{2}e^{j2(2\pi/6)t} + \frac{1}{2}e^{-j2(2\pi/6)t} - 2je^{j5(2\pi/6)t} + 2je^{-j5(2\pi/6)t} \end{aligned}$$

From this, we may conclude that the fundamental frequency of $x(t)$ is $2\pi/6 = \pi/3$. The non-zero Fourier series coeffcients of $x(t)$ are:

$$a_0 = 2, \quad a_2 = a_{-2} = \frac{1}{2}, \quad a_5 = a_{-5}^{*} = -2j$$

3.4. Since $\omega_0 = \pi$, $T = 2\pi/\omega_0 = 2$. Therefore,

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}dt$$

Now,

$$a_0 = \frac{1}{2}\int_0^1 1.5dt - \frac{1}{2}\int_1^2 1.5dt = 0$$

and for $k \neq 0$

$$\begin{aligned} a_k &= \frac{1}{2}\int_0^1 1.5e^{-jk\pi t}dt - \frac{1}{2}\int_1^2 1.5e^{-jk\pi t}dt \\ &= \frac{3}{2k\pi j}[1 - e{-}jk\pi] \\ &= \frac{3}{k\pi}e^{-jk(\pi/2)}\sin(\frac{k\pi}{2}) \end{aligned}$$

3.13. Let us first evaluate the Fourier series coefficients of $x(t)$. Clearly, since $x(t)$ is real and odd, a_k is purely imaginary and odd. Therefore, $a_0 = 0$. Now,

$$\begin{aligned} a_k &= \frac{1}{8}\int_0^8 x(t)e^{-j(2\pi/8)kt}dt \\ &= \frac{1}{8}\int_0^4 e^{-j(2\pi/8)kt}dt - \frac{1}{8}\int_4^8 e^{-j(2\pi/8)kt}dt \\ &= \frac{1}{j\pi k}[1 - e^{-j\pi k}] \end{aligned}$$

Clearly, the above expression evaluates to zero for all even values of k. Therefore,

$$a_k = \begin{cases} 0, & k = 0, \pm 2, \pm 4, \cdots \\ \frac{2}{j\pi k}, & k = \pm 1, \pm 3, \pm 5, \cdots \end{cases}$$

When $x(t)$ is passed through an LTI system with frequency response $H(j\omega)$, the output $y(t)$ is given by (see Section 3.8)

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0)e^{jk\omega_0 t}$$

where $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{4}$. Since a_k is non zero only for odd values of k, we need to evaluate the above summation only for odd k. Furthermore, note that

$$H(jk\omega_0) = H(jk(\pi/4)) = \frac{\sin(k\pi)}{k(\pi/4)}$$

is always zero for odd values of k. Therefore,

$$y(t) = 0.$$

3.20. (a) Current through the capacitor $= C\frac{dx(t)}{dt}$.
Voltage across resistor $= RC\frac{dy(t)}{dt}$.
Voltage across inductor $= LC\frac{d^2y(t)}{dt^2}$.
Input voltage = Voltage across resistor + Voltage across inductor + Voltage across capacitor.
Therefore,

$$x(t) = LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t)$$

Substituting for R, L and C, we have

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = x(t)$$

(b) We will now use an approach similar to the one used in part (b) of the previous problem. If we assume that the input is of the form $e^{j\omega t}$, then the output will be of the form $H(j\omega)e^{j\omega t}$. Substituting in the above differential equation and simplifying, we obtain

$$H(j\omega) = \frac{1}{-\omega^2 + j\omega + 1}$$

(c) The signal $x(t)$ is periodic with period 2π. Since $x(t)$ can be expressed in the form

$$x(t) = \frac{1}{2j}e^{j(2\pi/2\pi)t} - \frac{1}{2j}e^{-j(2\pi/2\pi)t},$$

the non-zero Fourier series coefficients of $x(t)$ are

$$a_1 = a_{-1}^* = \frac{1}{2j}.$$

Using the results derived in Section 3.8 (see eq.(3.124)), we have

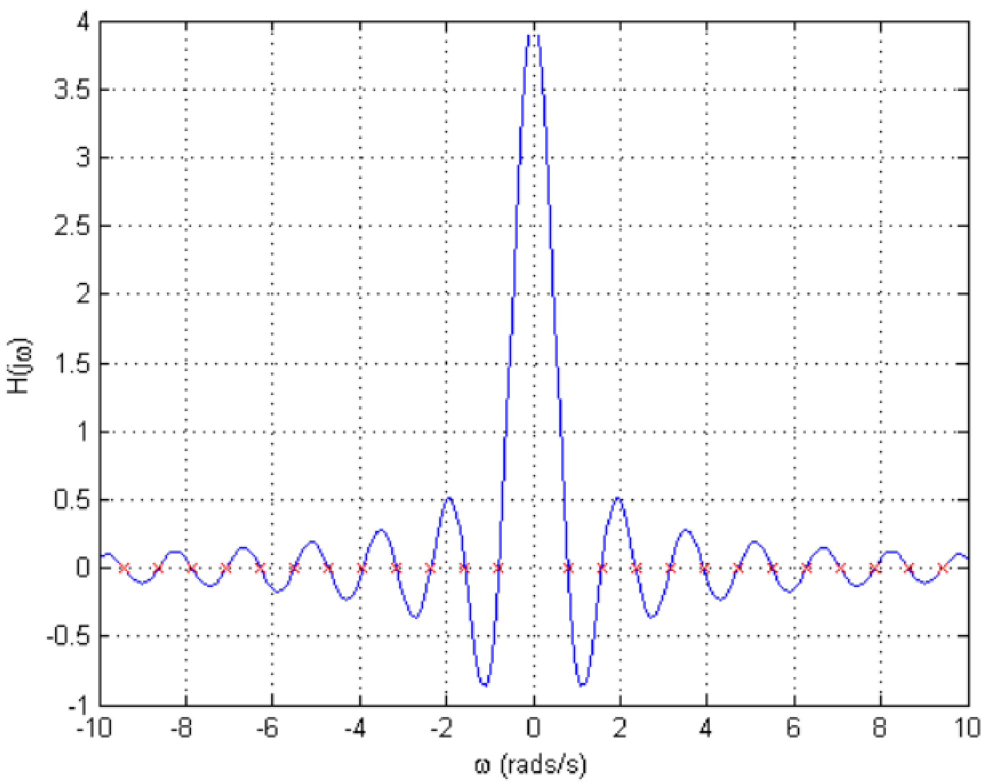
$$\begin{aligned} y(t) &= a_1H(j)e^{jt} - a_{-1}H(-j)e^{-jt} \\ &= (1/2j)(\frac{1}{j}e^{jt} - \frac{1}{-j}e^{-jt}) \\ &= (-1/2)(e^{jt} + e^{-jt}) \\ &= -\cos(t) \end{aligned}$$

3.22. (a) (i) $T = 1$, $a_0 = 0$, $a_k = \frac{j(-1)^k}{k\pi}$, $k \neq 0$.

(vi) $T = 4$, $\omega_0 = \pi/2$, $a_0 = 3/4$ and

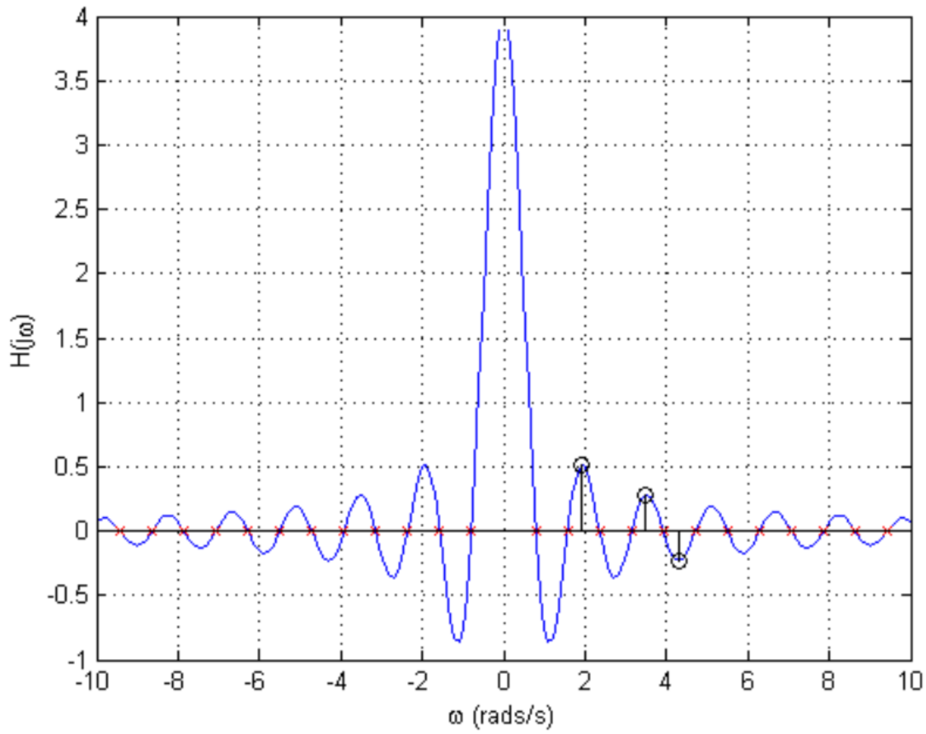
$$a_k = \frac{e^{-jk\pi/2}\sin(k\pi/2) + e^{-jk\pi/4}\sin(k\pi/4)}{k\pi}, \qquad \forall k.$$

```
%Software Problem 1

omega = [-10:0.1:10]; % range of freqs
H = sin(4*omega)./omega; % freq resp
plot(omega,H);
ylabel('H(j\omega)');
xlabel('\omega (rads/s)');

k = [-12:12];
omega_0 = 2*pi/8; % fundamental
k_omega_0 = k*omega_0; % harmonics
H_k_omega_0 = sin(4*k_omega_0)./k_omega_0; % freq resp @ harmonics
hold on;
plot(k_omega_0,H_k_omega_0,'rX');
hold off;
grid
```



Yes, the frequency response at the harmonic frequencies agree with my result in 3.13.

2) Software problem 2:



Below is the table where column 1 is the list of ω and column 2 is the list of the corresponding H(jω):

ω	H(jω)
0	4.0000
1.9000	0.5094
3.5000	0.2830
4.3000	-0.2318