Fall 2015 – EE312

Homework #4 Solution

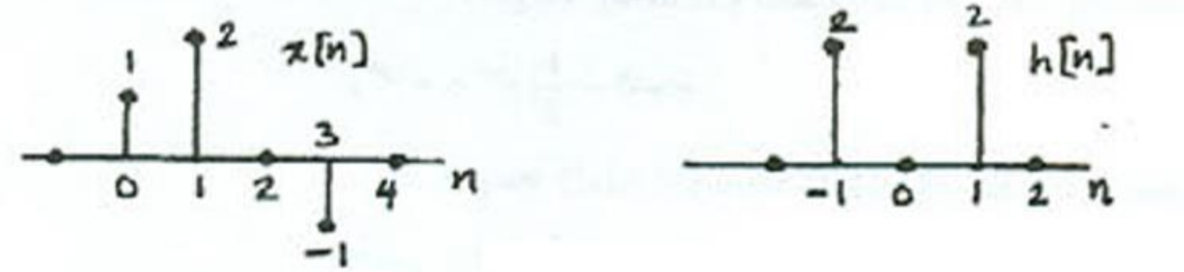
Textbook Problem Solution:

2.1:

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

The signals $x[n]$ and $h[n]$ are as shown



From this figure we can see that,

y2[n]=h[-1]x[1-k] +h[1]x[3-k];

So,

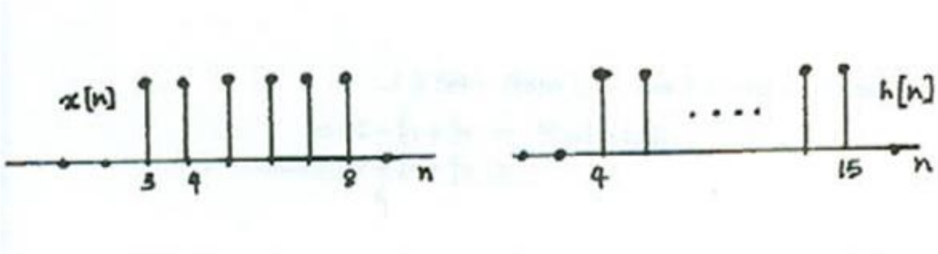
y2[n]=2δ[n+3]+4 δ[n+2] +2 δ[n+1]+2 δ[n] - 2 δ[n-2]

2.4)

We know that

$$y[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

The signals $x[n]$ and $y[n]$ are as shown in Figure .



From the above figure we see that the above summation reduces to:

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$

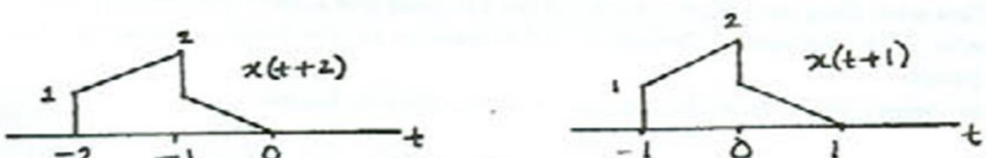
2.8. Using the convolution integral,

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral reduces to

$$x(t) * y(t) = x(t+2) + 2x(t+1)$$

The signals $x(t+2)$ and $2x(t+1)$ are plotted in Figure



Figure

Using these plots, we can easily show that

$$y(t) = \begin{cases} t+3, & -2 < t \le -1 \\ t+4, & -1 < t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

2.11. (a) From the given information, we see that $h(t)$ is non zero only for $0 \leq t \leq \infty$. Therefore,

$$\begin{aligned} y(t) &= x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \\ &= \int_{0}^{\infty} e^{-3\tau}(u(t-\tau-3) - u(t-\tau-5))d\tau \end{aligned}$$

We can easily show that $(u(t-\tau-3) - u(t-\tau-5))$ is non zero only in the range $(t-5) < \tau < (t-3)$. Therefore, for $t \leq 3$, the above integral evaluates to zero. For $3 < t \leq 5$, the above integral is

$$y(t) = \int_{0}^{t-3} e^{-3\tau} d\tau = \frac{1 - e^{-3(t-3)}}{3}$$

For $t > 5$, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} d\tau = \frac{(1 - e^{-6})e^{-3(t-5)}}{3}$$

Therefore, the result of this convolution may be expressed as

$$y(t) = \begin{cases} 0, & -\infty < t \leq 3 \\ \frac{1-e^{-3(t-3)}}{3}, & 3 < t \leq 5 \\ \frac{(1-e^{-6})e^{-3(t-5)}}{3}, & 5 < t \leq \infty \end{cases}$$

2.14. (a) We first determine if $h_1(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_1(\tau)|d\tau = \int_{0}^{\infty} e^{-t}d\tau = 1$$

Therefore, $h_1(t)$ is the impulse response of a stable LTI system.

(b) We determine if $h_2(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_2(\tau)|d\tau = \int_{0}^{\infty} e^{-t}|\cos(2t)|d\tau$$

This integral is clearly finite-valued because $e^{-t}|\cos(2t)|$ is an exponentially decaying function in the range $0 \leq t \leq \infty$. Therefore, $h_2(t)$ is the impulse response of a stable LTI system.

2.18. Since the system is causal, $y[n] = 0$ for $n < 1$. Now,

$$
\begin{aligned}
y[1] &= \frac{1}{4}y[0] + x[1] = 0 + 1 = 1 \\
y[2] &= \frac{1}{4}y[1] + x[2] = \frac{1}{4} + 0 = \frac{1}{4} \\
y[3] &= \frac{1}{4}y[2] + x[3] = \frac{1}{16} + 0 = \frac{1}{16} \\
&\vdots \\
y[m] &= \left(\frac{1}{4}\right)^{m-1} \\
&\vdots
\end{aligned}
$$

Therefore,

$$y[n] = \left(\frac{1}{4}\right)^{n-1} u[n-1]$$

2.21 (b):

y[n]=x[n]*h[n]

$y[n]=\sum_{k=0}^{n} x[k]h[n-k]$

$y[n]=\alpha^n[\ \sum_{k=0}^{n} 1\ ]u[n]$

$y[n]=(n+1)\ \alpha^n u[n]$

2.22 (c):

(c) The desired convolution is

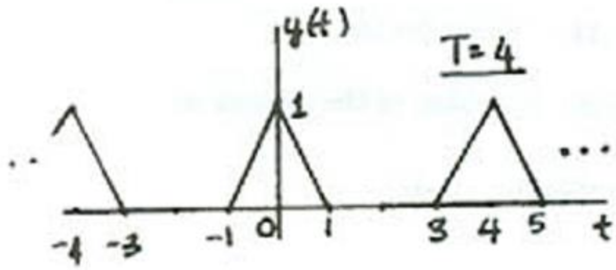
$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{0}^{2} \sin(\pi\tau)h(t-\tau)d\tau.
\end{aligned}
$$

This gives us

$$
y(t) = \begin{cases} 0, & t < 1 \\ (2/\pi)[1 - \cos\{\pi(t-1)\}], & 1 < t < 3 \\ (2/\pi)[\cos\{\pi(t-3)\} - 1], & 3 < t < 5 \\ 0, & 5 < t \end{cases}
$$

2.23 (a):

T=4



2.28 :

(b) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n=-2}^{\infty}(0.8)^n = 5 < \infty$.

(d) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n=-\infty}^{3} 5^n = \frac{625}{4} < \infty$

2.29:

(b) Not causal because $h(t) \neq 0$ for $t < 0$. Unstable because $\int_{-\infty}^{\infty}|h(t)| = \infty$.

(d) Not causal because $h(t) \neq 0$ for $t < 0$. Stable because $\int_{-\infty}^{\infty}|h(t)|dt = e^{-2}/2 < \infty$.

2.30. We need to find the output of the system when the input is $x[n] = \delta[n]$. Since we are asked to assume initial rest, we may conclude that $y[n] = 0$ for $n < 0$. Now,

$$y[n] = x[n] - 2y[n-1].$$

Therefore,

$$y[0] = x[0] - 2y[-1] = 1, \quad y[1] = x[1] - 2y[0] = -2, \quad y[2] = x[2] + 2y[2] = -4$$

and so on. In closed form,

$$y[n] = (-2)^n u[n].$$

This is the impulse response of the system.

2.31. Initial rest implies that $y[n] = 0$ for $n < -2$. Now

$$y[n] = x[n] + 2x[n-2] - 2y[n-1].$$

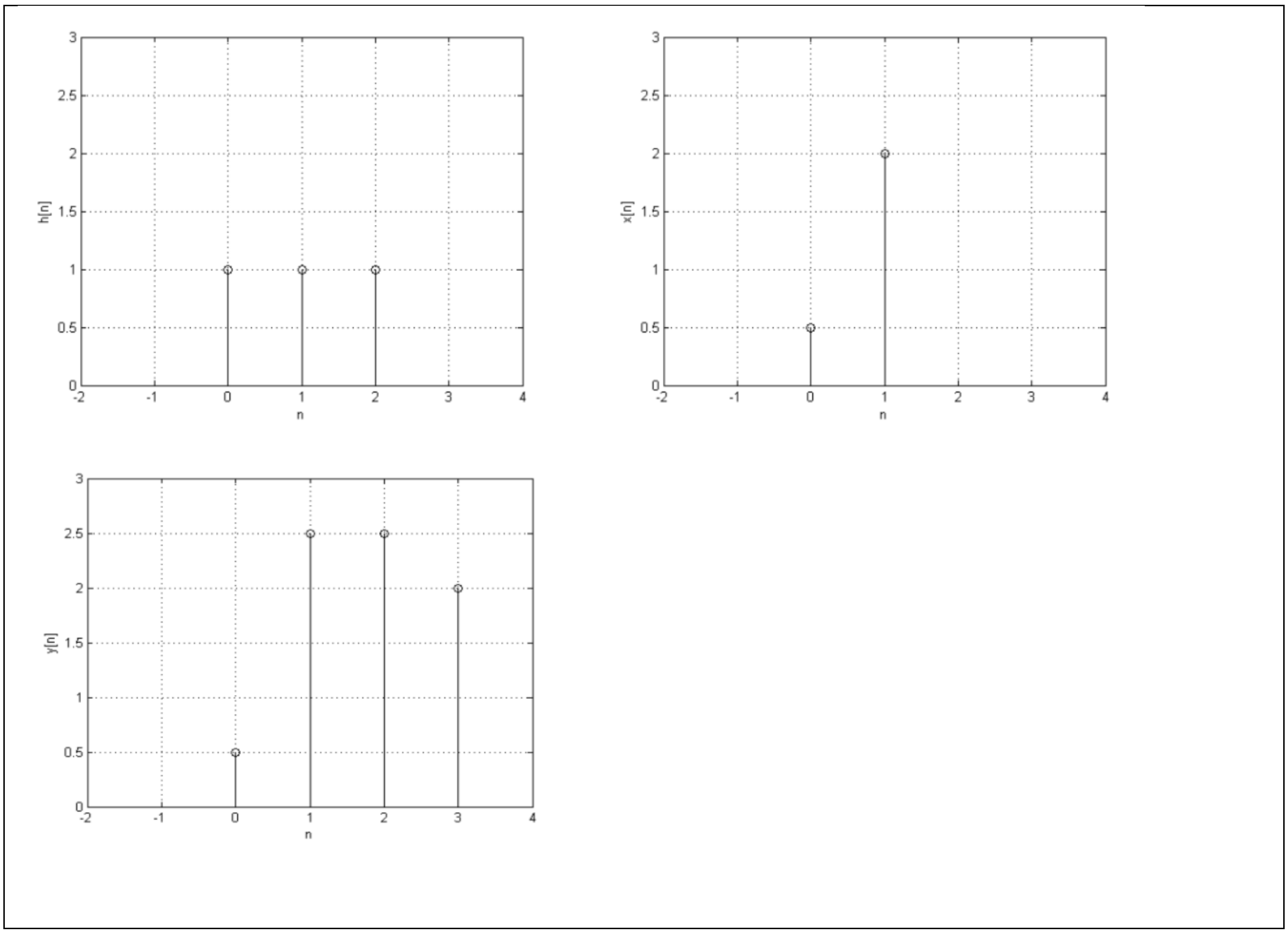
Therefore,

$$y[-2] = 1, \quad y[-1] = 0, \quad y[0]$$

$y[4] = 56, y[5] = -110, \quad y[n] = -110(-2)^{n-5}$ forn ≥ 5.

Software Problem Solution:

1:



2. The given impulse response has a unit pulse at n=0 and 1/4 pulse at n = 8000; for fs = 16000 there are 7999 zeros between these pulses. Thus the result of the convolution is to output the speech signal without delay or scaling as well as a 1/4-scaled copy with a delay of 1/2 s. These two are superimposed to produce the echoed signal.