Homework 10 Solution

5.19. (a) Taking the Fourier transform of both sides of the difference equation, we have

$$Y(e^{j\omega})\left[1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-2j\omega}\right]=X(e^{j\omega}).$$

Therefore,

$$H(ej\omega)=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-2j\omega}}=\frac{1}{(1-\frac{1}{2}e^{-j\omega})(1+\frac{1}{3}e^{-j\omega})}.$$

(b) Using Partial fraction expansion,

$$H(ej\omega)=\frac{3/5}{1-\frac{1}{2}e^{-j\omega}}+\frac{2/5}{1+\frac{1}{3}e^{-j\omega}}.$$

Using Table 5.2, and taking the inverse Fourier trasform, we obtain

$$h[n]=\frac{3}{5}\left(\frac{1}{2}\right)^n u[n]+\frac{2}{5}\left(-\frac{1}{3}\right)^n u[n].$$

5.22. (a) Using the Fourier transform synthesis eq. (5.8), we obtain

$$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{-3\pi/4}^{-\pi/4} e^{j\omega n}d\omega+\frac{1}{2\pi}\int_{\pi/4}^{3\pi/4} e^{j\omega n}d\omega \\ &= \frac{1}{\pi n}[\sin(3\pi n/4)-\sin(\pi n/4)] \end{aligned}$$

(c) Using the Fourier transform synthesis eq. (5.8), we obtain

$$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega/2}e^{j\omega n}d\omega \\ &= \frac{(-1)^{n+1}}{\pi(n-\frac{1}{2})} \end{aligned}$$

(f) The given Fourier transform may be written as

$$\begin{aligned} X(e^{j\omega}) &= e^{-j\omega}\sum_{n=0}^{\infty}(1/5)^n e^{-j\omega n}-(1/5)\sum_{n=0}^{\infty}(1/5)^n e^{-j\omega n} \\ &= 5\sum_{n=1}^{\infty}(1/5)^n e^{-j\omega n}-(1/5)\sum_{n=0}^{\infty}(1/5)^n e^{-j\omega n} \end{aligned}$$

Comparing each of the two terms in the right-hand side of the above equation with the Fourier transform analysis eq. (5.9) we obtain

$$x[n]=\left(\frac{1}{5}\right)^{n-1}u[n-1]-\left(\frac{1}{5}\right)^{n+1}u[n].$$

5.29. (a) Let the output of the system be $y[n]$. We know that

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}).$$

In this part of the problem

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}.$$

(i) We have

$$X(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}e^{-j\omega}}.$$

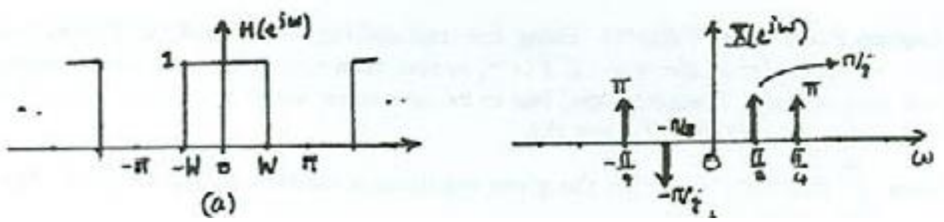
Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{1 - \frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{-2}{1 - \frac{1}{2}e^{-j\omega}} + \frac{3}{1 - \frac{3}{4}e^{-j\omega}} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

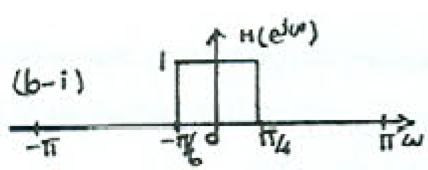
$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n].$$

5.30. (a) The frequency response of the system is as shown in Figure S5.30.



(b) The Fourier transform $X(e^{j\omega})$ of $x[n]$ is as shown in Figure S5.30.

(i) The frequency response $H(e^{j\omega})$ is as shown in Figure S5.30. Therefore, $y[n] = \sin(\pi n/8)$.



5.33. (a) Taking the Fourier transform of the given difference equation we have

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 + \frac{1}{2}e^{-j\omega}}.$$

(b) The Fourier transform of the output will be $Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$.

(i) In this case

$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}.$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{1 + \frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{1/2}{1 - \frac{1}{2}e^{-j\omega}} + \frac{1/2}{1 + \frac{1}{2}e^{-j\omega}} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(-\frac{1}{2}\right)^n u[n].$$

(c) (i) We have

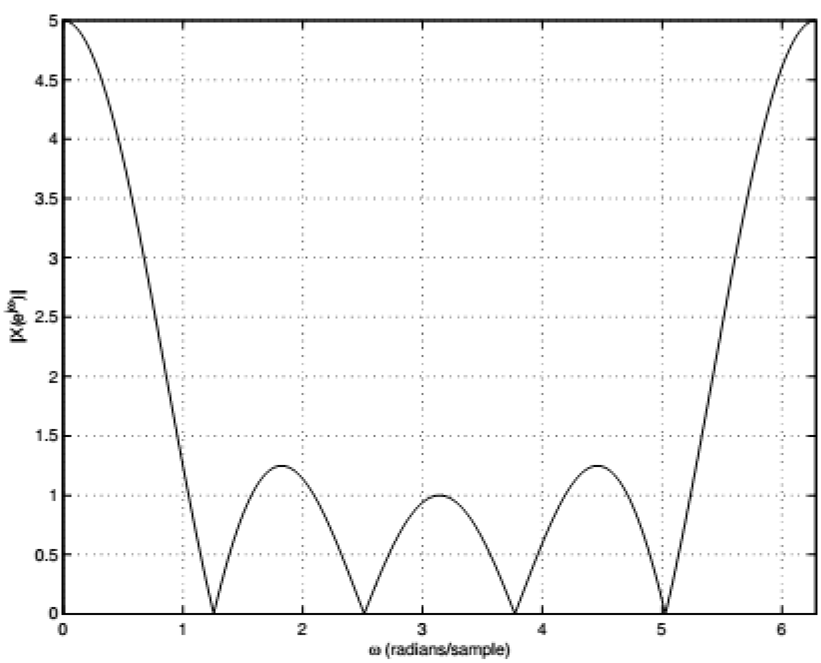
$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1 - \frac{1}{4}e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{1 + \frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{1}{(1 + \frac{1}{2}e^{-j\omega})^2} - \frac{\frac{1}{4}e^{-j\omega}}{(1 + \frac{1}{2}e^{-j\omega})^2} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

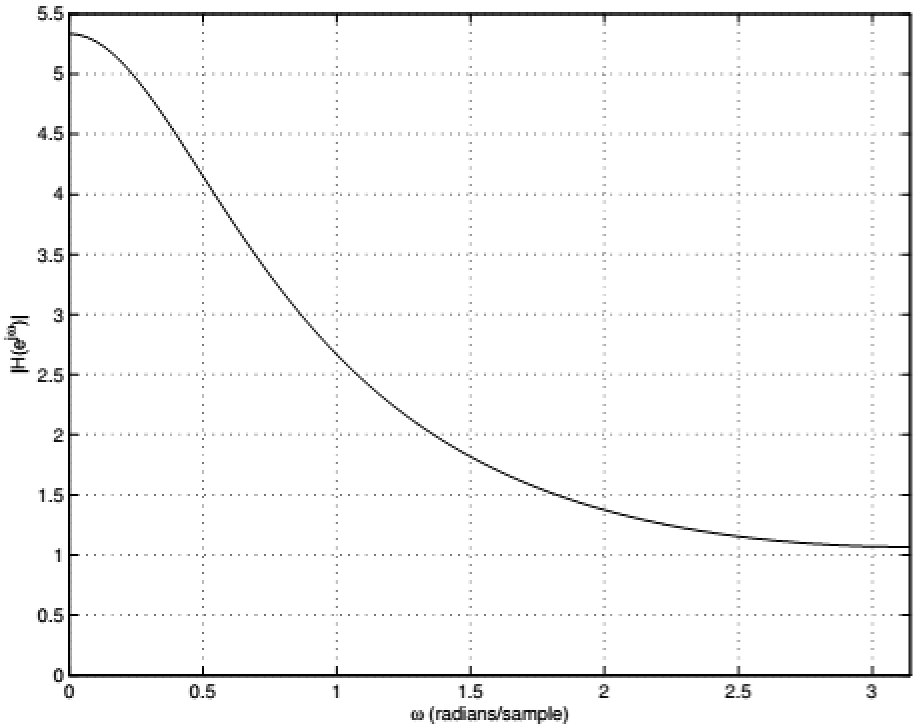
$$y[n] = (n+1)\left(-\frac{1}{2}\right)^n u[n] - \frac{1}{4}n\left(-\frac{1}{2}\right)^{n-1} u[n-1].$$

Software Problem Solution:

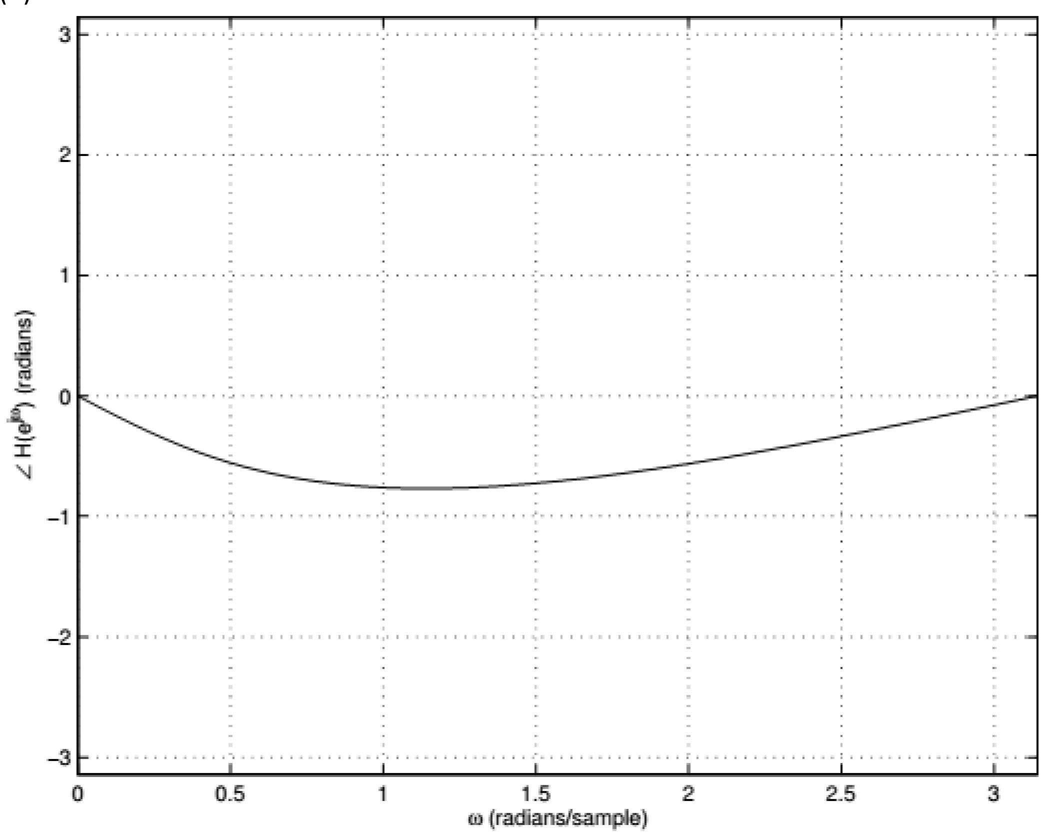
1.



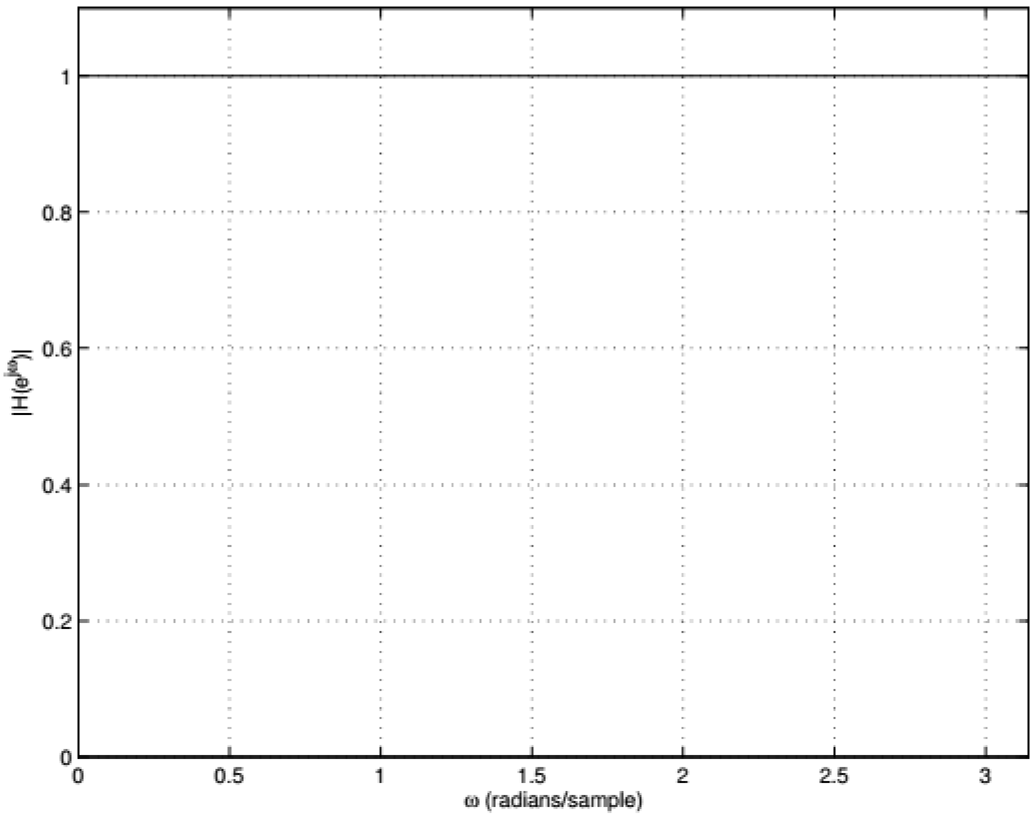
2. (a)


5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
|H(e^jω)|
0
0.5
1
1.5
2
2.5
3
ω (radians/sample)

(b)

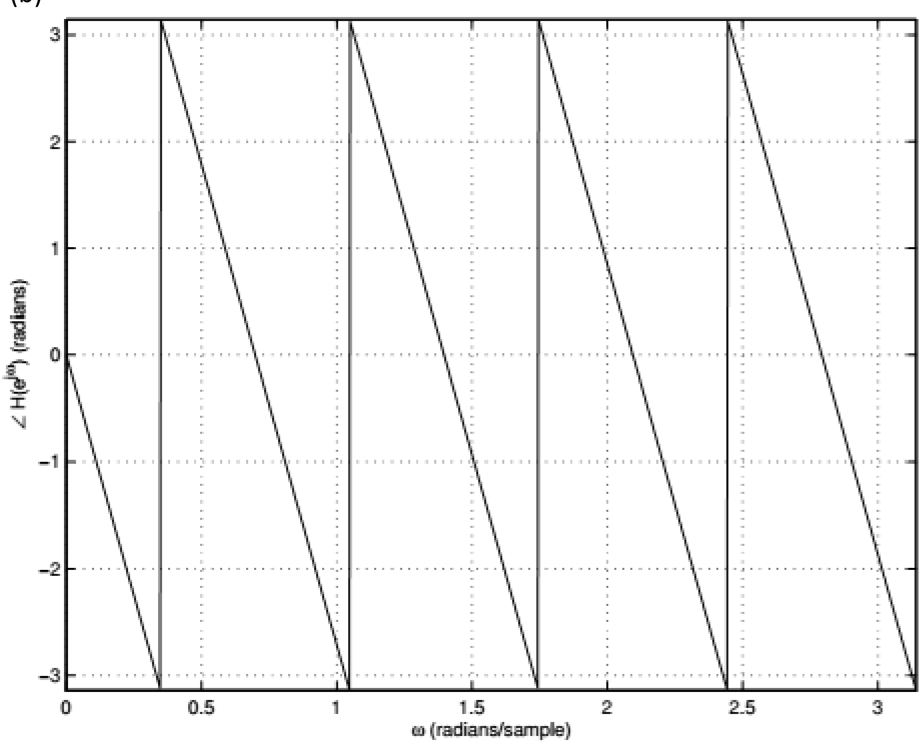

3
2
1
0
−1
−2
−3
∠H(e^jω) (radians)
0
0.5
1
1.5
2
2.5
3
ω (radians/sample)

3. (a)



1
0.8
0.6
0.4
0.2
0
|H(e^{jω})|
0
0.5
1
1.5
2
2.5
3
ω (radians/sample)

(b)



3
2
1
0
−1
−2
−3
∠ H(e^{jω}) (radians)
0
0.5
1
1.5
2
2.5
3
ω (radians/sample)