1) $\int_0^t T \, dT$

$= \left. \frac{T^2}{2} \right|_0^t$

$= \frac{t^2}{2}$

2) $\int_0^1 t e^{-t/2} \, dt$

$= \left. \frac{t e^{-t/2}}{-1/2} - \frac{e^{-t/2}}{(-1/2)^2} \right|_0^1$

$= \left. -2te^{-t/2} - 4e^{-t/2} \right|_0^1$

$= -2e^{-1/2} - 4e^{-1/2} + 4$

$= 4 - 6e^{-1/2}$

$= 0.3608$

3) $\int_0^\infty e^{-t/2} \, dt$

$= \left. \frac{e^{-t/2}}{(-1/2)} \right|_0^\infty$

$= 2$

4) $$\int_{-\pi}^{\pi} e^{-jk\omega t}\, dt$$

$$= \frac{-1}{jk\omega} e^{-jk\omega t} \Big|_{-\pi}^{\pi}$$

$$= \frac{e^{-jk\omega\pi} - e^{jk\omega\pi}}{-jk\omega}$$

$$= \frac{e^{jk\omega\pi} - e^{-jk\omega\pi}}{jk\omega} \times \frac{2}{2}$$

$$= \frac{2}{k\omega} \sin(k\omega\pi)$$

5) $$\lim_{x \to 0} \frac{e^{x} - x - 1}{x^2}$$ by L'Hospital rule

$$= \lim_{x \to 0} \frac{e^{x} - 1}{2x}$$ by L'Hospital rule

$$= \lim_{x \to 0} \frac{e^{x}}{2}$$

$$= \frac{1}{2}$$

6) $\lim_{x \to \pi/2} \frac{\cos(x)}{\pi - 2x}$ by L'Hospital rule

$$= \lim_{x \to \pi/2} \frac{-\sin(x)}{-2}$$

$$= \frac{1}{2}$$

7) Let $S = \sum_{n=0}^{\infty} \alpha^n = 1 + \alpha + \alpha^2 + \alpha^3 + \dots$

then $\alpha S = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \dots$

$$S - \alpha S = 1$$

$$S = \frac{1}{1-\alpha}$$

8) for $\alpha = 1$

$$\sum_{n=0}^{N-1} \alpha^n = \underbrace{1 + 1 + 1 - \cdots\cdots + 1}_{N \text{ times}}$$

$$= \underline{\underline{N}}$$

for $\alpha \neq 1$

$$(1 - \alpha^N) = (1-\alpha)(1 + \alpha + \alpha^2 + \cdots + \alpha^{N-1})$$

$$\frac{(1-\alpha^N)}{(1-\alpha)} = (1 + \alpha + \alpha^2 + \cdots + \alpha^{N-1})$$

$$\underline{\underline{\frac{1-\alpha^N}{1-\alpha} = \sum_{n=0}^{N-1} \alpha^n}}$$

9) $\underline{\underline{A = 1}}$, $\underline{\underline{B = -1}}$

10) $\underline{\underline{A = -1}}$, $\underline{\underline{B = 2}}$

11) $\frac{dy}{\partial x} + 2y = e^x$

let, $y = y_H + y_P$ (Homogeneous & particular solution)

Guess, $y_H = Ae^{sx}$ then,

$\frac{d}{\partial x} Ae^{sx} + 2Ae^{sx} = 0$

$Ase^{sx} + 2Ae^{sx} = 0$

$sy_x + 2y_H = 0$

$sy_H = -2y_H$

$\Rightarrow s = -2$

So $\underline{\underline{y_H = Ae^{-2x}}}$

Again, Gues, $y_P = Be^x$ then,

$\frac{\partial}{\partial x} Be^x + 2Be^x = e^x$

$Be^x + 2Be^x = e^x$

$3Be^x = e^x$

$3y_P = e^x$

$\underline{\underline{y_P = \frac{1}{3} e^x}}$

So Total Soln

$y = y_H + y_P$

$y = Ae^{-2x} + 1/3\, e^x$

when $x=0$, $y=1$. So

$1 = A + 1/3$

$\underline{\underline{A = 2/3}}$

Solution is $\underline{\underline{y = \frac{2}{3} e^{-2x} + \frac{1}{3} e^x}}$

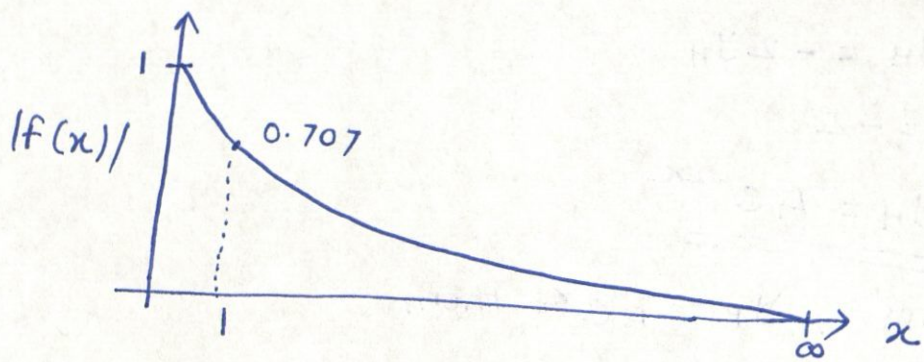
12) $f(x) = \dfrac{1}{1+jx}$

We have to draw $|f(x)|$ vs x

$|f(0)| = 1$

$|f(1)| = \dfrac{1}{\sqrt{2}} = 0.707$

$|f(\infty)| = 0$

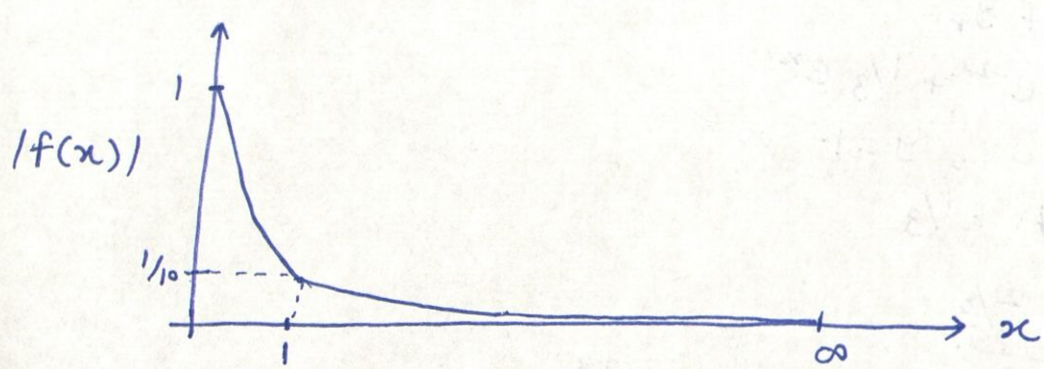


13) $f(x) = \dfrac{1}{1+j10x}$

We have to draw $|f(x)|$ vs. x

$|f(0)| = 1$

$|f(1)| = 1/10$, $|f(\infty)| = 0$



14) $e^{j0} = 1$

$e^{j\pi/4} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$e^{j\pi/2} = j$

$e^{j3\pi/4} = \frac{-\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$e^{-j\pi} = -1$

$e^{j2\pi} = 1$

15)

$1 = e^{j0} = e^{j2\pi}$

$1 + j = \sqrt{2}\, e^{j\pi/4}$

$j = e^{j\pi/2}$

$1 - j = \sqrt{2}\, e^{-j\pi/4}$

$-1 = e^{j\pi} = e^{-j\pi}$

$-j = e^{-j\pi/2} = e^{j3\pi/2}$