Homework #7: Chapter 3 (due Oct. 16, 2015)

Preliminary

- Textbook reading Ch. 3.6 - 3.9 (pp. 211-239)
- As a reminder, EE312 office hours are on Wed. from 9:00-10:00am and Thu. from 3:00-4:00pm.
- Please direct all email to pdeleon@nmsu.edu (do not send email via Canvas). All requests for bonus points will receive a confirmation email within 48 hours.
- In order to receive full credit for homework problems, you must provide a detailed solution. Simply writing a few, summarized steps toward the answer will result in minimal credit.
- All problems are worth +10 points unless otherwise noted.

Textbook Problems

3.10	3.14	3.16(b)	3.21
3.28(a) Figure P.3.28(a)	3.29(a),(c)	3.33(a)	

Hints:

3.33 Take the LCCDE and use the eigenfunction theory to form

$$\frac{d}{dt}\left[H(j\omega)e^{j\omega t}\right] + 4\left[H(j\omega)e^{j\omega t}\right] = e^{j\omega t}.$$

Solve for $H(j\omega)$ to get the frequency response of the CT system and use this to complete the problem.

Software Problems

Read the next section beginning on p. 2, on using MATLAB to compute the Fourier Series (FS) and Discrete-Time Fourier Series (DTFS) of periodic signals.

1. Write a MATLAB code to compute and plot the DTFS coefficients of

$$x[n] = 1 - \cos(3\pi n/8).$$

2. Write a MATLAB code to compute and plot the DTFS coefficients for the DT periodic square wave given on p. 218 Figure 3.16 of the textbook. Assume $N_1 = 2$ and $N = 10$. Your plot should match Figure 3.17(a) for the range $0 \leq k \leq 9$.

3. A DT, periodic signal $x[n]$ is real-valued and has a fundamental period of $N = 9$. The nonzero FS coefficients for $x[n]$ are $a_0 = 2$, $a_1 = a_{-1}^* = e^{j\pi/4}$, $a_3 = a_{-3}^* = 2e^{j\pi/2}$, $a_4 = a_{-4}^* = 3e^{j\pi}$. Write a MATLAB code to synthesize and plot $x[n]$.

Discrete-Time Fourier Series

The discrete-time Fourier series (DTFS) is a representation for periodic, DT sequences using complex exponentials as the basis functions. For a signal $x[n]$ with period of N, the DTFS synthesis and analysis equations are given by

$$x[n] \quad = \quad \sum_{k=0}^{N-1} a_k e^{jk(2\pi/N)n} \tag{1}$$

and

$$a_k \quad = \quad \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n} \tag{2}$$

respectively. Although the above equations index the summation from 0 to $N-1$, any consecutive N integers may be used due to the periodicity of both $x[n]$ and a_k. Let

$$\mathbf{x} \quad = \quad [x[0],\ x[1],\ \ldots,\ x[N-1]]^T \tag{3}$$

be an N-point vector containing the samples over one period. The DTFS of $\mathbf{x}$ can be computed in MATLAB with the command

```
>> a = (1/N)*fft(x)
```

where the result is the N-point vector

$$\mathbf{a} \quad = \quad [a_0,\ a_1,\ \ldots,\ a_{N-1}]^T\,. \tag{4}$$

Similarly given a vector $\mathbf{a}$ containing the DTFS coefficients, the inverse DTFS can be computed in MATLAB with the command

```
>> x = N*ifft(a)
```

where the result is the N-point vector

$$\mathbf{x} \quad = \quad [x[0],\ x[1],\ \ldots,\ x[N-1]]^T\,. \tag{5}$$

Fourier Series

The Fourier series (FS) is a representation for periodic, continuous-time signals using complex exponentials as the basis functions. For a signal $x(t)$ with period of T, the FS synthesis and analysis equations are given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{6}$$

and

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt \tag{7}$$

respectively. Since MATLAB is executed on a digital computer, we can only numerically evaluate or approximate the integral required for computing the FS coefficients. Furthermore, there may be an infinite number of FS coefficients but we can only compute a finite number of them. Therefore, we assume a vector of N sample points with sample spacing Δ of $x(t)$ over one period

$$\mathbf{x} \;=\; \left[x(\Delta),\; x(2\Delta),\; \ldots,\; x(N\Delta)\right]^T \tag{8}$$

and computation of K FS coefficients. The FS of $\mathbf{x}$ can be computed in MATLAB with the command

```
>> a = (1/T)*fft(x,K)
```

where the result is the K-point vector

$$\mathbf{a} \;=\; \left[a_0,\; a_1,\; \ldots,\; a_{K-1}\right]^T . \tag{9}$$

Similarly given a vector $\mathbf{a}$ containing the FS coefficients, the inverse FS can be computed in MATLAB with the command

```
>> x = T*ifft(a,N)
```

where the result is the N-point vector

$$\mathbf{x} \;=\; \left[x[0],\; x[1],\; \ldots,\; x[N-1]\right]^T \tag{10}$$

of sample points of $x(t)$. If Δ is small and the values of K and N are very large, then MATLAB's approximations are very close to the actual values.

Roundoff Errors

For some signals, $x(t)$ and $x[n]$, the Fourier series coefficients are real-valued. However, when numerically computed, there may be a very small roundoff error leading to a very small ($< 10^{-16}$) imaginary part on the coefficients. In these cases you may simply take the real part of the coefficients.