New Mexico State University
Klipsch School of Electrical Engineering

EE312 - Signals and Systems I
Spring 2010
Final Exam

Name: ____________________

Solve Probs. 1 and 2 and any two from Probs. 3–6.
Circle below which of Probs. 3–6 you want graded.

Prob. 1		/ 25 points
Prob. 2		/ 25 points
Prob. 3		/ 25 points
Prob. 4		/ 25 points
Prob. 5		/ 25 points
Prob. 6		/ 25 points
Total		/ 100 points

Prob. 1

(i) Consider a continuous-time (CT) complex exponential signal, $x_0(t) = e^{j\omega_0 t}$.

(a) TRUE / FALSE : $x_0(t)$ may be aperiodic but it depends on ω_0.

(b) TRUE / FALSE : The oscillation rate of $x_0(t)$ always increases as ω_0 increases.

(c) TRUE / FALSE : The signal $x_1(t) = e^{j\omega_1 t}$ can be the same as $x_0(t)$ even if $\omega_1 \neq \omega_0$.

(ii) A Discrete-Time (DT) system is described by the input/output rule,

$$y[n] = x[4n+1].$$

Circle which system properties hold.

(a) Linear (b) Time-Invariant (c) Causal

(d) BIBO Stable (e) Memoryless

(iii) Consider the periodic signal $x(t) = 1 + \cos(20\pi t)$. Circle the Fourier Series (FS) coefficients. If the coefficient is not listed, you may assume it is equal to zero.

(a) $a_0 = 1,\ a_1 = 1,\ a_{-1} = -1$ (b) $a_0 = 1,\ a_{10} = 1,\ a_{-10} = 1$

(c) $a_0 = 1,\ a_1 = 1/2,\ a_{-1} = 1/2$ (d) $a_0 = 1,\ a_{20} = 1/2,\ a_{-20} = 1/2$

(iv) Which equation is the inverse transform of the Discrete-Time Fourier Transform (DTFT) $X(e^{j\omega})$?

(a) $x[n] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$ (b) $x[n] = \frac{1}{2\pi}\int_{0}^{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$

(c) $x[n] = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$ (d) $x[n] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(e^{j\omega})e^{j\omega n}d\omega$

Prob. 1 (cont.)

(v) Consider a causal, stable CT system described by the following frequency response

$$H(j\omega) = \frac{1 + j\omega}{1 + j2\omega}.$$

Which differential equation equivalently describes the system? Assume initial rest conditions.

(a) $y(t) + 2\frac{dy(t)}{dt} = x(t) + \frac{dx(t)}{dt}$

(c) $y(t) + \frac{dy(t)}{dt} = x(t) + 2\frac{dx(t)}{dt}$

(b) $y(t) + \frac{d^2y(t)}{dt^2} = x(t) + \frac{dx(t)}{dt}$

(d) $y(t) + \frac{dy(t)}{dt} = x(t) + \frac{d^2x(t)}{dt^2}$

(vi) The transfer function of a *stable*, linear time-invariant (LTI) system is $H(s) = s/[(s+1)(s-1)]$. The Region of Convergence (ROC) is

(a) $\Re\{s\} < -1$

(b) $\Re\{s\} > 0$

(c) $\Re\{s\} > 1$

(d) $-1 < \Re\{s\} < 1$

(vii) The transfer function of a *stable*, LTI system is $H(s) = s/[(s+1)(s-1)]$. The input to the system is $x(t) = e^{(1+j)t}$. The output of the system is

(a) $y(t) = \lambda e^{-(1+j)t}$ where $\lambda = H(1+j)$

(b) $y(t) = \lambda e^{t} e^{jt}$ where $\lambda = H(1)H(j)$

(c) $y(t) = \lambda e^{(1+j)t}$ where $\lambda = H(1+j)$

(d) $y(t) = \lambda e^{-(1+j)t}$ where $\lambda = H(-1-j)$

(viii) The transfer function of a *stable*, LTI system is $H(s) = s/[(s+1)(s-1)]$. This system is

(a) Causal

(b) Anti-Causal

(c) Mixed (has both causal and anti-causal parts)

(d) None of the above

Prob. 2

The transfer function of a *causal*, LTI system is given by

$$H(s) = \frac{s}{s^3 + 3s^2 + 4s + 2} = \frac{s}{(s+1)(s^2+2s+2)} = \frac{s}{(s+1)(s+1+j)(s+1-j)} \quad (2.1)$$

(a) Determine the ROC of $H(s)$. Plot the pole-zero pattern for $H(s)$ and shade the ROC on your plot.

(b) If the ROC includes the $j\omega$-axis, determine the frequency response of the system $H(j\omega)$. If the ROC does not include the $j\omega$-axis write "no $H(j\omega)$."

(c) Determine the Linear Constant-Coefficient Differential Equation (LCCDE) for the system.

Prob. 2 (cont.)

(d) Sketch a block diagram for the system. Your diagram can only include elements such as adders, multiplications by coefficients, and integrators. Do not use differentiators in your diagram.

(e) Determine the impulse response $h(t)$ for the system.

Prob. 3

(a) Consider a causal, CT system described by

$$\frac{dy(t)}{dt} + 4y(t) = x(t) \tag{3.1}$$

with initial rest conditions and an input signal composed of two complex exponential signals

$$x(t) = \frac{1}{2}e^{j2t} + \frac{1}{3}e^{j3t}. \tag{3.2}$$

Use *eigenfunction* theory to determine the output signal, $y(t)$.

(b) Consider a DT system with unit-impulse response

$$h[n] = \left(-\frac{1}{4}\right)^n u[n] \tag{3.3}$$

and an input signal composed of two complex exponential signals

$$x[n] = 1 + e^{j\pi n/2} + e^{j\pi n}. \tag{3.4}$$

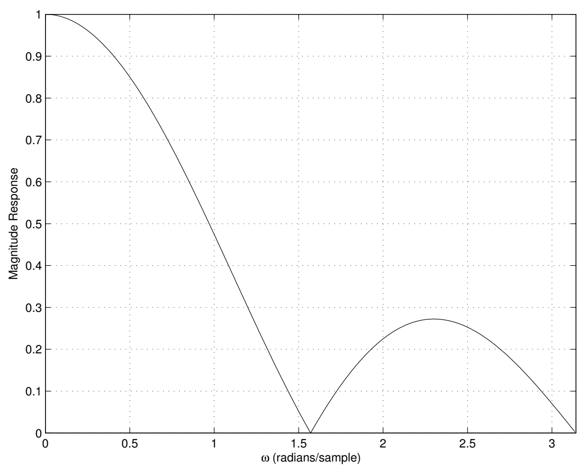
Use *eigenfunction* theory to determine the output signal, $y[n]$.

Prob. 4

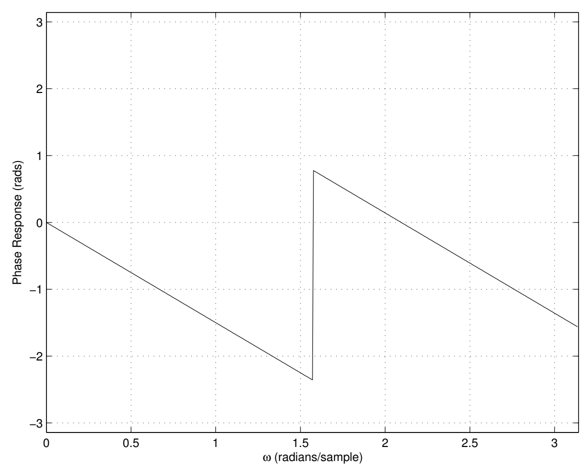
A DT filter has impulse response

$$h[n] \quad = \quad \frac{1}{4}\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\right). \tag{4.1}$$

The frequency response $H(e^{j\omega})$ is shown below where (a) is the magnitude response $|H(e^{j\omega})|$ and (b) is the phase response $\angle H(e^{j\omega})$. Note that $H(e^{j\omega}) = |H(e^{j\omega})|e^{j\angle H(e^{j\omega})}$.



(a)



(b)

(a) Using the graphs determine $|H(e^{j\omega_0})|$ and $\angle H(e^{j\omega_0})$ for $\omega_0 = 2.1$ rad/sample.

Hint: Read the values off the graphs.

(b) Determine the output signal $y[n]$ if the input signal $x[n] = e^{j2.1n}$.

(c) Determine the output signal $y[n]$ if the input signal $x[n] = \cos(2.1n)$.

Prob. 5

The following problems must be solved by *explicitly* convolving $x(t)$ and $h(t)$ or $x[n]$ and $h[n]$. You may not solve these problems via transform-domain techniques.

(a) Let $x(t) = u(t+1) - u(t-1)$ and $h(t) = u(t-2) - u(t-4)$. Determine $y(t) = h(t) * x(t)$.

(b) Let $x[n] = u[n+1] - u[n-1]$ and $h[n] = \frac{1}{2}\delta[n+1] + \delta[n] + \frac{1}{2}\delta[n-1]$. Determine $y[n] = h[n] * x[n]$ for $n = -2, 0, 2$.

Prob. 5 (cont.)

(c) Let $x(t) = e^{-3t}u(t)$ and $h(t) = e^{-5t}u(t)$. Determine $y(t) = h(t) * x(t)$.

Prob. 6

For the following problems, you may use any method including basic FS/DTFS/FT/DTFT pairs and properties, direct expansion into complex exponentials (where applicable), or direct calculation.

(a) A periodic ($N = 3$) DT signal has samples $x[8] = 1$, $x[9] = 2$, $x[10] = 3$. Determine the Discrete-Time Fourier Series (DTFS) coefficients.

(b) Determine the DTFT of the signal, $x[n] = u[n-2] - u[n-6]$.

Prob. 6 (cont.)

(c) Let

$$x_1(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (6.1)$$

and $x_2(t) = x_1(t-1)$. Compute the Fourier Transform (FT) of $x(t) = x_1(t) + x_2(t)$.