# New Mexico State University
# Klipsch School of Electrical Engineering

# EE312 - Signals and Systems I
# Fall 2015
# Final Exam

## Name: ____________________

**Solve problems 1–3 and two from problems 4 – 7.**
**Circle below which two of problems 4 – 7 you wish to be graded.**

| | |
|---|---|
| Prob. 1 | / 20 points |
| Prob. 2 | / 20 points |
| Prob. 3 | / 20 points |
| Prob. 4 | / 20 points |
| Prob. 5 | / 20 points |
| Prob. 6 | / 20 points |
| Prob. 7 | / 20 points |
| Total | / 100 points |

# Prob. 1

The linear, constant-coefficient differential equation (LCCDE) of a continuous-time (CT), linear, time-invariant (LTI) system is given by

$$\frac{1}{10000}\frac{d^2y(t)}{d^2t} + \frac{101}{1000}\frac{dy(t)}{dt} + y(t) = \frac{1}{100}\frac{dx(t)}{dt} + x(t).$$

Assume at rest conditions.

(a) Draw a block diagram of the system using differentiators and not integrators.

(b) Determine the frequency response, $H(j\omega)$ of the system.

# Prob. 1 (cont.)

(c) Determine the impulse response, $h(t)$ of the system.

(d) Let the input to the system be, $x(t) = e^{j100t}$. Use any method* you wish to determine the output, $y(t)$.

* Solve LCCDE for $y(t)$, convolve $h(t) * x(t)$, transform-domain $\mathcal{F}^{-1}\left\{H(j\omega)X(j\omega)\right\}$, or eigenfunction theory

# Prob. 2

The linear, constant-coefficient difference equation (LCCDE) of a discrete-time (DT) LTI system is given by

$$y[n] - \frac{1}{4}y[n-2] = x[n] - 2x[n-1].$$

Assume at rest conditions.

(a) Draw a block diagram of the system.

(b) Determine the frequency response, $H(e^{j\omega})$ of the system.

# Prob. 2 (cont.)

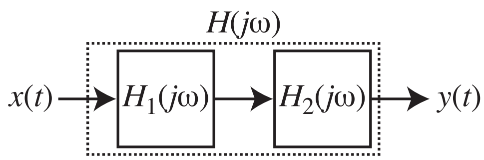
(c) Determine the impulse response, $h[n]$ of the system.

(d) Let the input to the system be, $x[n] = e^{j2n}$. Use any method* you wish to determine the output, $y[n]$.

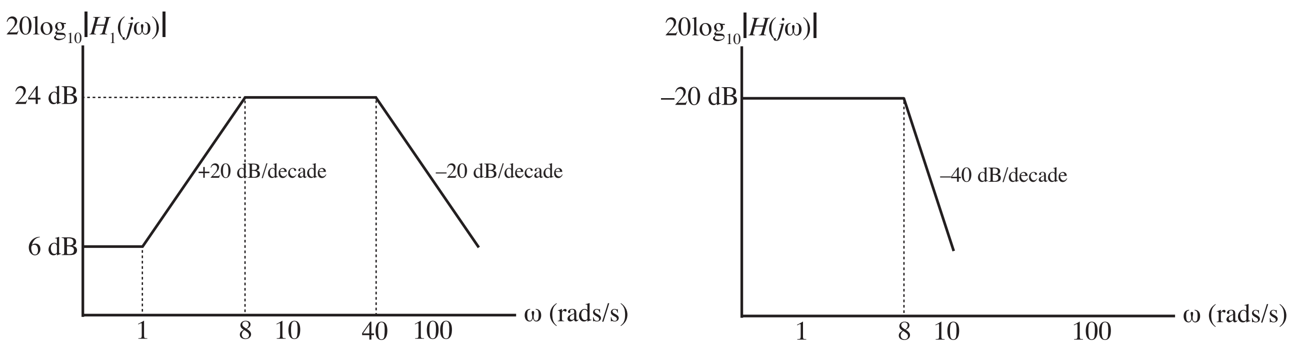
* Solve LCCDE for $y[n]$, convolve $h[n] * x[n]$, transform-domain $\mathcal{F}^{-1}\left\{H(e^{j\omega})X(e^{j\omega})\right\}$, or eigenfunction theory

# Prob. 3

A CT LTI system with frequency response, $H(j\omega)$ is constructed by cascading two CT LTI systems with frequency responses, $H_1(j\omega)$ and $H_2(j\omega)$ as depicted below; obviously $H(j\omega) = H_1(j\omega)H_2(j\omega)$.




The two figures below show the straight-line approximations of the Bode magnitude plots of $H_1(j\omega)$ and $H(j\omega)$. Complete the following parts to determine $H_2(j\omega)$.




(a) $H_1(j\omega)$ (b) $H(j\omega)$

(a) Determine the three break frequencies of $H_1(j\omega)$.

# Prob. 3 (cont.)

(b) Determine the constant factor for $H_1(j\omega)$, $A_1$ such that

$$H_1(j\omega) = \frac{A_1(j\omega + \omega_1)}{(j\omega + \omega_2)(j\omega + \omega_3)}$$

where $\omega_1$, $\omega_2$, and $\omega_3$ are the break frequencies from (a).

Hint: From the plot, $20\log_{10}|H_1(j0)| = 6$ or $|H_1(j0)| = 2$.

(c) Determine the break frequency(ies) and constant factor for $H(j\omega)$.

(d) Determine $H_2(j\omega) = H(j\omega)/H_1(j\omega)$ using your results in (b) and (c).

# Prob. 4

**Part I:**

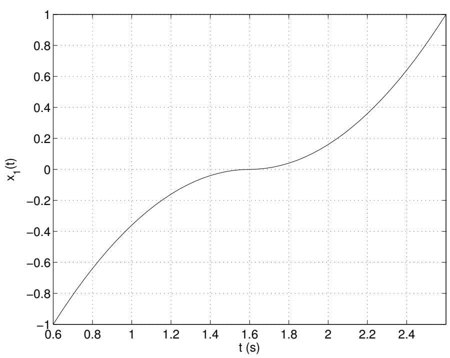
Let

$$x(t) = \begin{cases} t^2, & 0 \le t < 1 \\ -t^2, & -1 \le t < 0 \\ 0, & \text{otherwise} \end{cases} \quad \overset{\mathcal{F}}{\leftrightarrow} \quad X(j\omega)$$

For this problem, the actual value of $X(j\omega)$ does not matter. The first solution is given as an example.

(a) Graph $x_1(t) = x(t - 1.6)$ and write $X_1(j\omega)$ in terms of $X(j\omega)$.

Solution: $X_1(j\omega) = e^{-j1.6\omega} X(j\omega)$




(b) Graph $x_1(t) = x\left(-\frac{1}{2}t\right)$ and write $X_1(j\omega)$ in terms of $X(j\omega)$.

(c) Graph $x_1(t) = x\left(3t + 1\right)$ and write $X_1(j\omega)$ in terms of $X(j\omega)$.

# Prob. 4 (cont.)

**Part II:**

Let

$$x[n] = \frac{\sin(\pi n/4)}{\pi n} \quad \overset{\mathcal{F}}{\leftrightarrow} \quad X(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \pi/4 \\ 0, & \pi/4 < |\omega| \le \pi \end{cases}$$

$X(e^{j\omega})$ is periodic with a period of $2\pi$.

(d) $x_1[n] = x[n-3]$. Graph the magnitude and phase response of $X_1(e^{j\omega})$ for $-\pi \le \omega \le \pi$.

(e) $x_1[n] = x[n]{*}e^{j\pi n/2}$. Graph the magnitude and phase response of $X_1(e^{j\omega})$ for $-\pi \le \omega \le \pi$.

(f) $X_1(e^{j\omega}) = e^{-j4\omega}X(e^{j\omega})$. Write $x_1[n]$ in terms of $x[n]$ and graph the magnitude and phase response of $X_1(e^{j\omega})$ for $-\pi \le \omega \le \pi$.

# Prob. 5

(a) Let $x(t) = u(t)$ and $h(t) = e^{-1000t}u(t) + e^{-10t}u(t)$. Determine $y(t) = h(t) * x(t)$ for $-\infty < t < \infty$ using graphical convolution(s).

# Prob. 5 (cont.)

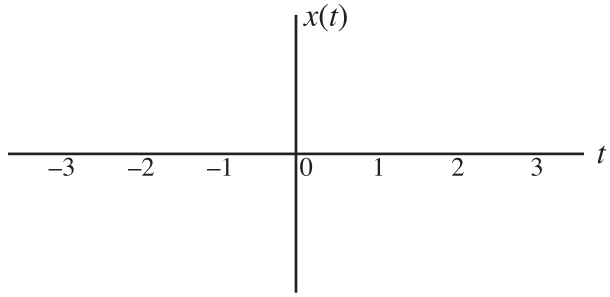
(b) Let $x[n] = u[n]$ and $h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n]$. Determine $y[n] = h[n] * x[n]$ for $-\infty < n < \infty$ using graphical convolution(s).

# Prob. 6

(a) Let

$$x(t) = \begin{cases} 1 - e^{-t}, & -1 \leq t < 0 \\ e^t - 1, & 0 \leq t < 1 \end{cases}$$

be a periodic signal with a fundamental period $T = 2$ s. Graph $x(t)$ over the interval $-3 \leq t \leq 3$ and determine the Fourier Series (FS) coefficients, $a_k$.



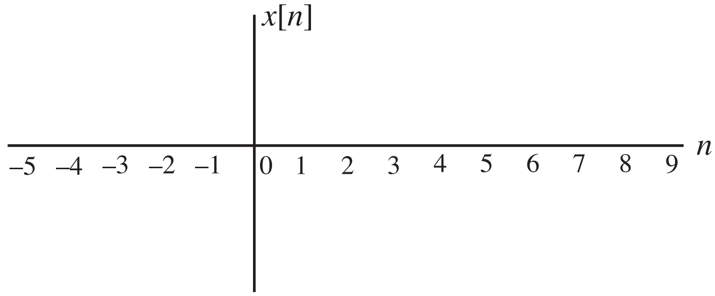

$a_k = ?$

# Prob. 6 (cont.)

(b) Let

$$x[n] = \delta[n] - 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-3] + \delta[n-4]$$

be a periodic signal with a fundamental period $N = 5$. Graph $x(t)$ over the interval $-5 \leq n \leq 9$ and determine the Discrete-Time Fourier Series (DTFS) coefficients, $a_k$.




$a_k = ?$

# Prob. 7

Let the input signal be

$$x(t) = \cos(20\pi t) + \cos(200\pi t) + \cos(2000\pi t)$$

and the impulse response be

$$h(t) = 10000te^{-100t}u(t).$$

Use eigenfunction theory (not phasors or convolution) to determine the output signal, $y(t)$. Express $y(t)$ in terms of cosines.