NAME: ____ GRADE: ______________

EE492 / EE592 Real-Time Digital Signal Processing
Quiz #3
Open Books, Notes, Calculators (no programs, no graphing)

Each question is worth 10 points unless otherwise noted.

1. The *minimum* mean-squared error (MMSE), $\xi_{\min}$ is the resulting mean-squared error (MSE), ξ when the adaptive filter coefficients, $\hat{\mathbf{h}}$ are optimal. Show that

$$\xi_{\min} = E\left[y^2(n)\right] - \mathbf{p}^T\mathbf{R}^{-1}\mathbf{p}.$$

Note: For two matrices, $\mathbf{A}$ and $\mathbf{B}$ of compatible dimensions, $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$ where ^{T} denotes matrix transpose.

2. We have argued that the Hilbert filter may be responsible for poor ANC performance when the reference tone has a very low frequency or a frequency close to the Nyquist frequency. What was the argument?

3. In obtaining the LMS algorithm from the steepest-descent algorithm, we used instantaneous estimates of the autocorrelation matrix, $\mathbf{R}$ and the cross-correlation vector, $\mathbf{p}$. Why was this necessary?

Fill in the missing instructions and comments denoted with '?' for the following *non-real-time* codes (Questions 4 – 6). The first is done as an example.

4. Compute the following inner product

$$y = \mathbf{h}^T \mathbf{x} = \begin{bmatrix} 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$

using the following code:

```
        org   p:$100
        move  ?,?          ;?
        mpy   #0.2,x0,?    ;?
        move  ?,x0         ;?
        mac   #0.3,?,a     ;a = 0.4*0.2 + 0.3*0.1 (ANSWER IN a)
```

Solution:

```
        org   p:$100
        move  #0.4,x0      ;x0 = 0.4
        mpy   #0.2,x0,a    ;a = 0.4*0.2
        move  #0.1,x0      ;x0 = 0.1
        mac   #0.3,x0,a    ;a = 0.4*0.2 + 0.3*0.1 (ANSWER IN a)
```

5. Compute an estimate of the signal power

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N} x^2(n)$$

using the following code:

```
N      equ 8                     ;define number of samples in estimate

       org y:$0
x_vec  dc    0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8

       org p:$100
       move  ?,?                       ;point to x_vec
       clr   ?     ?,x0                ;clr ?, get x_1
       rep   #?
       mac   ?,?,?       ?:(r0)+,x0    ;accumulate x_n^2, get next x_n
       asr   ?,?,?                     ;divide by N, estimate in accumulator a
```

6. Compute the following linear combination of vectors

$$\mathbf{y} = \mathbf{h} + \mu\mathbf{x}$$
$$= \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix} + 0.5 \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$
$$= \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

using the following code:

```
mu     equ   0.5                ;define mu

       org ?:$0
x_vec ?      ?                  ;malloc for x_vec
       org ?:x_vec
       dc ?,?                   ;load values for x_vec
y_vec ?      ?                  ;malloc for y_vec

       org ?:$0
h_vec ?      ?                  ;malloc for h_vec
       org ?:h_vec
       dc ?,?                   ;load values for h_vec

       org p:$100
       move #h_vec,?
       move #x_vec,?
       move #y_vec,?

       move x:?,a         ;a = h_1
       move y:(r4)+,?     ;?
       macr ?,x0,a        ;?
       move a,?           ;y_1 = h_1 + mu*x_1

       move x:(r0),a      ;a = h_2
       move ?,?           ;x0 = x_2
       macr ?,x0,a        ;?
       move ?,y:(r5)      ;?
```

Write a code for the following *real-time* programming tasks (Questions 7 – 8), assuming the Modified Pass Pack beginning on p. 307 of the text. Identify by file name (PASS.ASM or PASS.DAT) and line number where you would place your instructions. The first programming task is done as an example.

7. Write a code which multiplies the right channel by $g = 0.7$.

Solution:

In PASS.DAT insert at line 11 the following line:

```
g   equ   0.7
```

In PASS.ASM insert at line 83 the following line:

```
   move   #g,x1   ; move g to x1
```

In PASS.ASM replace lines 98 – 100 with the following lines:

```
   mpyr   x0,x1,a ; multiply right sample by 0.7, rounded result in a
   move   a,x0    ; move result to x0 for move to TX_BUFF_BASE in loop_1
```

8. (30 points) Assume the sample $x(n)$, $y(n)$ come in on the right, left channel respectively; $\mu = 0.001$; and a length 32 adaptive filter. Write a generic code (not the ANC) which sends the output of the adaptive filter, $\hat{y}(n)$ to the left channel and then adjusts the adaptive filter using the LMS algorithm given in (6.21) on p. 182 similar to the block diagram on p. 178. Be sure you remember the details: allocating memory for arrays and variables, initializing the adaptive filter to zero, computing the filter output, computing the error, and adjusting the filter coefficients.

8. (continued)

EE592 Only

9. For the ANC of Project #2, we define $\mathbf{x}(n) = \left[x_0(n) \quad x_1(n) \right]^T$ (see p. 185 for the ANC block diagram). How do we know

$$0 \le \mathbf{x}(n)^T \mathbf{x}(n) = A^2 \le 1$$

where A is the amplitude of the reference tone? This inequality was used in determining the cases for the division required in NLMS. Note the inequality above is the corrected version.