Prob. 1

(a) See Prob. 1.1 solution attached at the end.

(b) The recreation of textbook Figure 1.4 is shown in Fig. 1. Actual values for w^* are shown in Table 1.

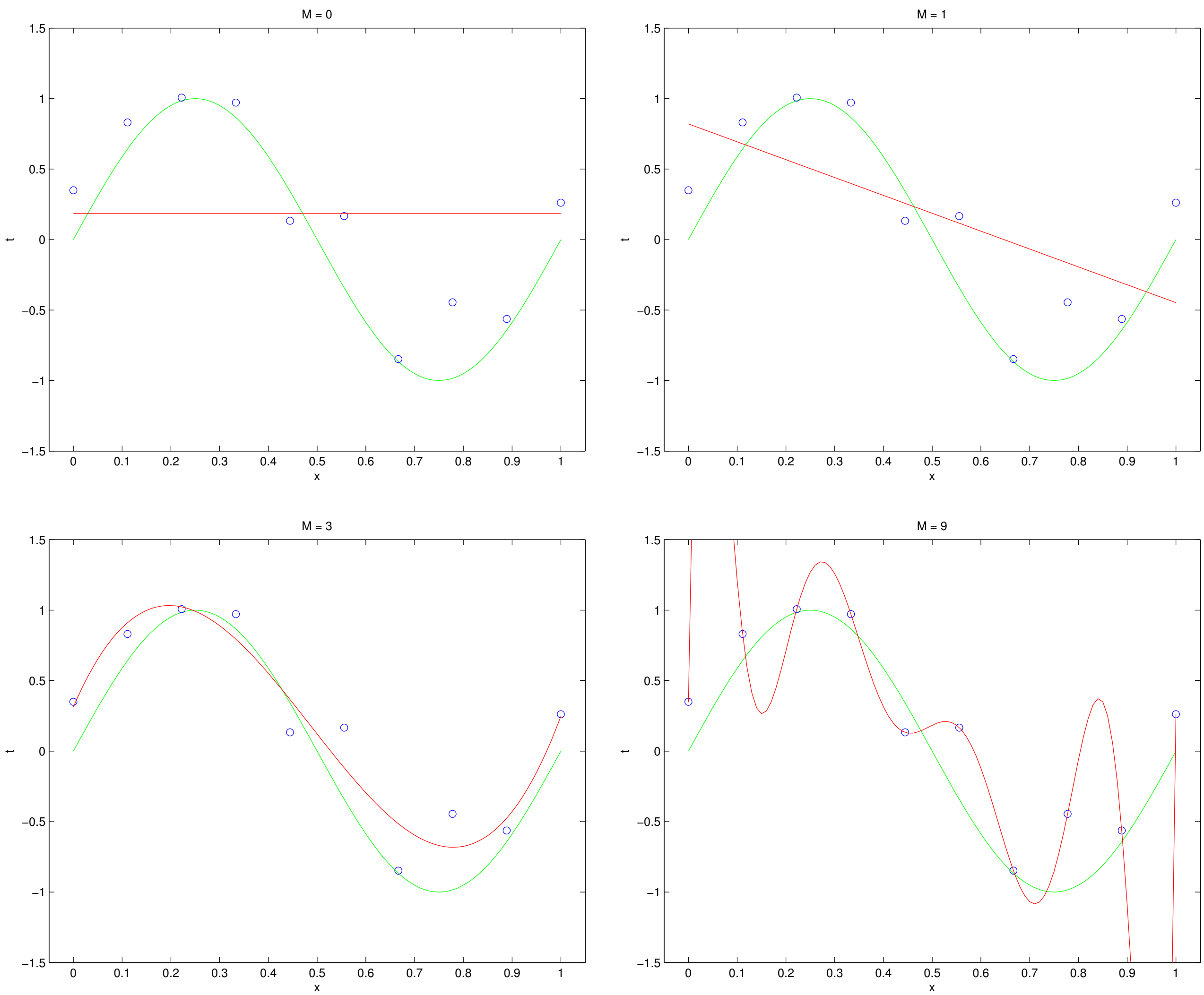

Figure 1: Recreation of Figure 1.4 for $M = 0, 1, 3,$ and 9.

(c) The recreation of textbook Figure 1.5 is shown in Fig. 2. The numbers match only if the factor of 2 is ommitted from the E_{RMS} formula in (1.3).

(d) The recreation of textbook Figure 1.6 is shown in Fig. 3.

Prob. 2

(a) See Prob. 1.2 solution attached at the end.

Table 1: Values for w*

	$M = 0$	$M = 1$	$M = 3$	$M = 9$
w_0^*	0.1863	0.8202	0.3137	0.3495
w_1^*		-1.2678	7.9854	232.3667
w_2^*			-25.4261	-5,321.7919
w_3^*			17.3741	48,568.0031
w_4^*				231,637.9222
w_5^*				640,038.6630
w_6^*				-1,061,794.8022
w_7^*				1,042,394.7290
w_8^*				557,680.1324
w_9^*				125,200.7991

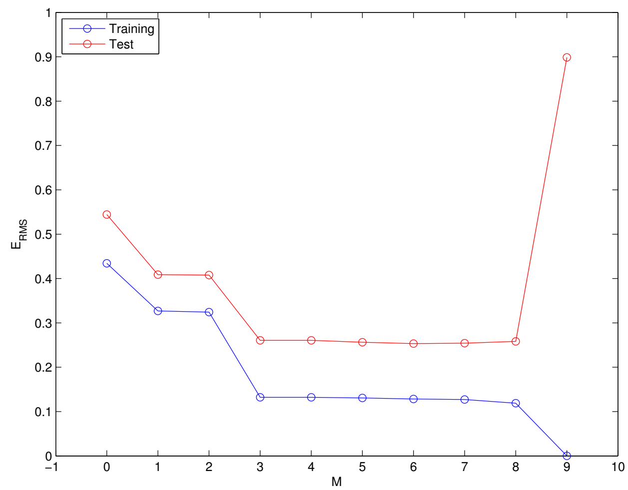

Figure 2: Recreation of Figure 1.5 RMS values for training and test data.

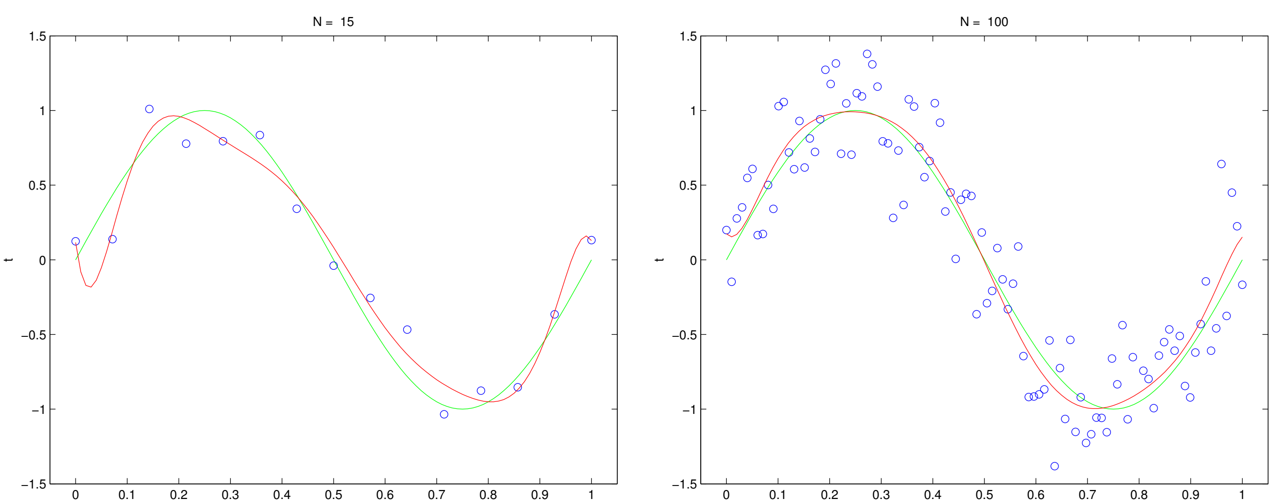

Figure 3: Recreation of Figure 1.6 for $M = 9$ and $N = 15,\ 100$.

(b) The recreation of textbook Figure 1.7 is shown in Fig. 4. Actual values for w* are shown in Table 2.

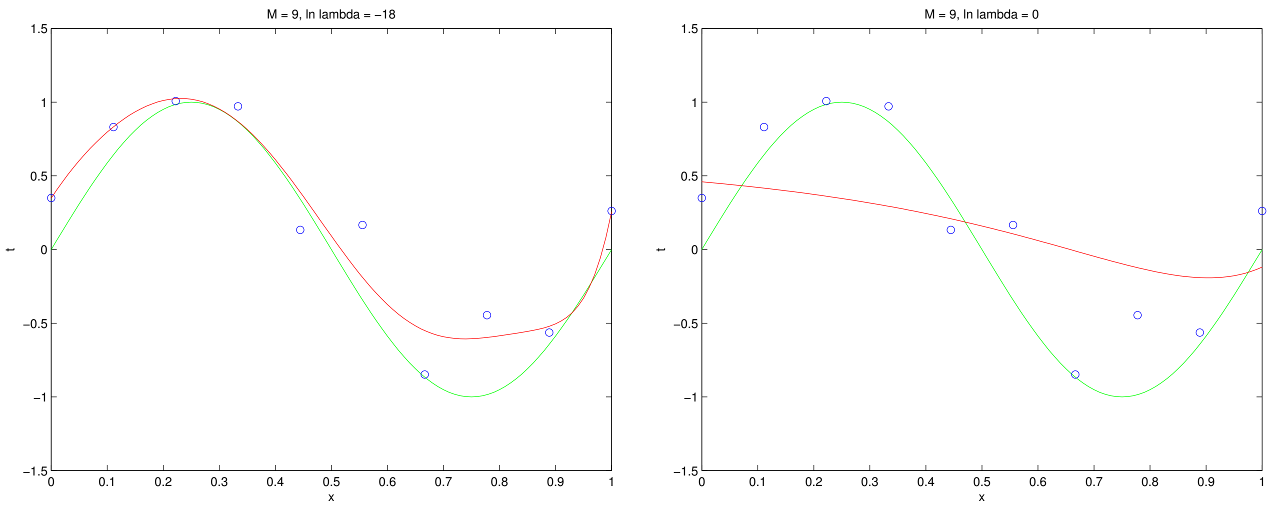

Figure 4: Recreation of Figure 1.7 for $\ln \lambda = -18,\ 0$.

Table 2: Values for w*

	$\ln \lambda = -\infty$	$\ln \lambda = -18$	$\ln \lambda = 0$
w_0^*	0.3496	0.3490	0.4597
w_1^*	232.7248	5.7684	-0.3449
w_2^*	-5,330.0353	-14.9653	-0.3655
w_3^*	48,642.0216	29.8572	-0.2449
w_4^*	-231,984.5571	-95.6202	-0.1230
w_5^*	640,980.1888	35.1719	-0.0235
w_6^*	-1,063,332.2058	168.7130	0.0538
w_7^*	1,043,882.1420	-18.0353	0.1135
w_8^*	-558,465.2928	-273.1754	0.1598
w_9^*	125,374.9248	162.1930	0.1962

(c) The recreation of textbook Figure 1.8 is shown in Fig. 5. The factor of 2 is ommitted from the E_{RMS} formula in (1.3).

Prob. 3

(a) See solution in online solution manual.

(b) Histograms of the sample mean and sample variance are shown in Figure 6. Estimate averages and other data are given in Table 3.

Prob. 4

(a) We use ML estimators for the distributional parameters of each class. See Table 4.

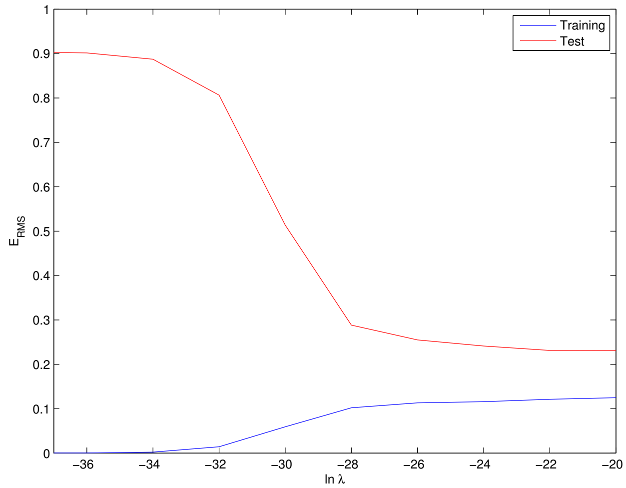

Figure 5: Recreation of Figure 1.8 RMS values for training and test data.

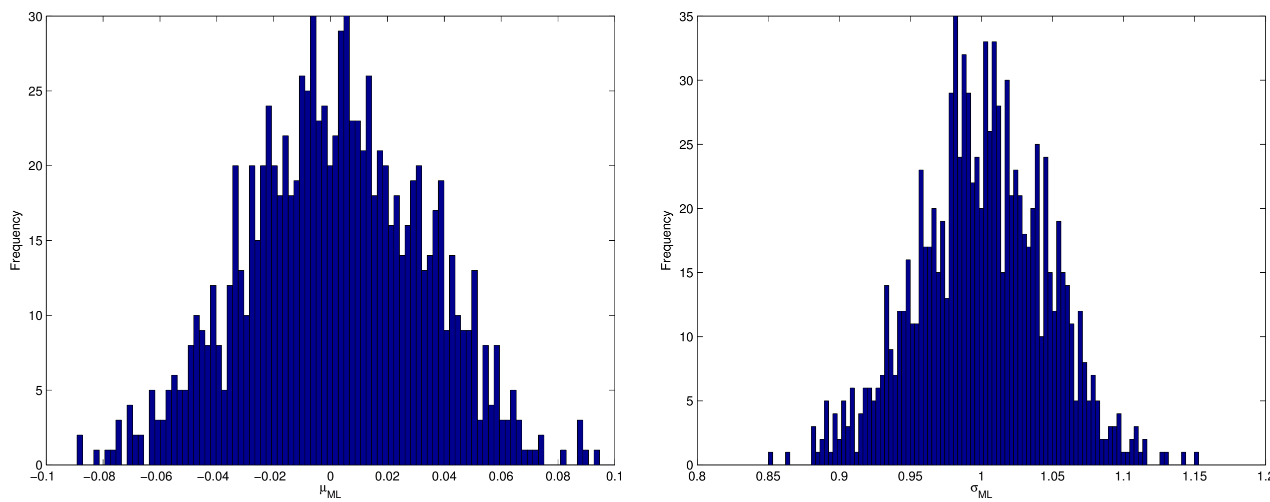

Figure 6: Histograms of the sample mean and sample variance.

Table 3: Maximum likelihood estimation of distributional parameters.

Actual Parameter	Estimate Average	Theoretical Parameter
$\mu = 0$	$\bar{\mu}_{\mathrm{ML}} = 0.0018$	$\mathbb{E}[\mu_{\mathrm{ML}}] = 0$
$\sigma^2 = 1$	$\bar{\sigma^2}_{\mathrm{ML}} = 1.0002$	$\mathbb{E}[\sigma^2_{\mathrm{ML}}] = 0.9990$

Table 4: Estimated normal distribution parameters for each class

Class	μ_{ML}	σ^2_{ML}
1	-0.0040	0.9217
2	4.0681	1.9961

(b) We set $p(x, \mathcal{C}_1) = p(x, \mathcal{C}_2)$ and solve for x (crossover or decision point). This leads to a quadratic

$$\left(-\frac{1}{2\sigma_1^2} + \frac{1}{2\sigma_2^2}\right)x^2 + \left(\frac{\mu_1}{\sigma_1^2} - \frac{\mu_2}{\sigma_2^2}\right)x + \left[\ln\left(\frac{\sigma_2}{\sigma_1}\right) - \frac{\mu_1^2}{2\sigma_1^2} + \frac{\mu_2^2}{2\sigma_2^2}\right] = 0$$

Solving the quadratic leads to the optimal crossover point $\hat{x} = 1.7707$. Plots of the distributions and optimal decision boundary are shown in Figure 7.

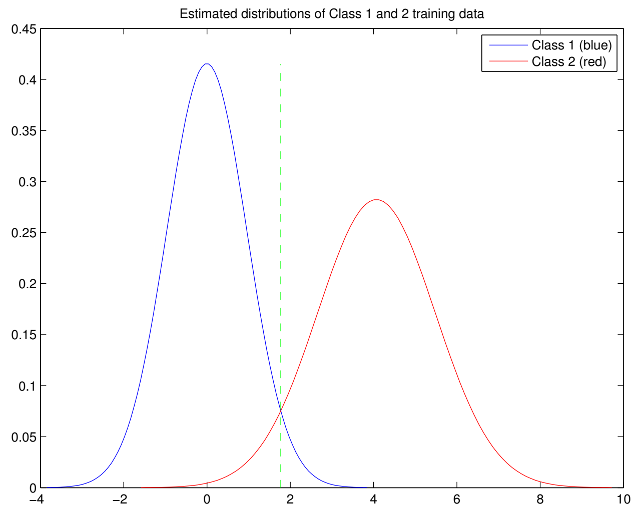

Figure 7: Distributions of class 1 and 2 training data and crossover point.

(c) Using the optimal decision boundary, the confusion matrix using the test set is shown below

```
confusion_matrix =
    0.9800    0.0200
    0.0400    0.9600
```

(d) We choose a sub-optimal decision boundary, $x_0 = 3.0$. The confusion matrix is shown below.

```
confusion_matrix =
    1.0000    0.0000
    0.1600    0.8400
```

(e) Using the MAP rule, we checked each of the test classifications and they were identical to those in (c). As expected the confusion matrix was identical

```
confusion_matrix_ML =
    0.9800    0.0200
    0.0400    0.9600
```