

1 Lecture Outline

Reading: Chapter 12 Interpolation, Decimation, and Oversampling

This lecture will cover the following topics

- Interpolation and Oversampling (Section 12.1)

2 Interpolation and Oversampling (Section 12.1)

There may be applications which require conversion of the sampling rate of a digital signal from one rate, f_s to another rate, f'_s . One obvious way to carry out this conversion is to reconstruct the analog signal and resample at the desired rate. This approach while requiring additional analog and digital hardware, may also introduce additional noise in the A/D and D/A stages. Ideally, we would like to convert sampling rates entirely in the digital domain.

Sampling rate *increase* is accomplished by interpolation, that is, the process of inserting additional samples between the original low-rate samples. The inserted, or interpolated, samples are calculated by an FIR digital filter.

Figure 1: Orfanidis, p. 632, Figure 12.1.1

From the figure, we see with respect to the fast time scale, the low-rate samples may be thought of as being separated by three zero samples. The 4-fold rate *expander* or *upsampler* simply inserts three zero samples for every low-rate sample. The job of the FIR filter is to replace the three zeros by the calculated interpolated values.

The interpolating filter is sometimes called an *oversampling digital filter* because it operates at the fast rate $f'_s = 4f_s$. However, because only one out of every four input samples is non-zero, the required filtering operations may be rearranged in such a way as to operate only on the low-rate samples, thus, effectively reducing the computational requirements of the filter by a factor of four in this case. This is accomplished by replacing the high-rate interpolating FIR filter by four shorter FIR subfilters, known as *polyphase* filters, each which operates at the low rate f_s and a multiplexer.

3 Oversampling for D/A

3.1 Conventional D/A

An interesting application of interpolation is the use of oversampling digital filters in D/A, where they help to alleviate the need for high-quality analog anti-image postfilters. To understand the motivation behind this application, consider an analog signal sampled at a rate f_s , such as 44.1 kHz for digital audio.

The analog signal is prefiltered by an analog lowpass antialiasing prefilter having cutoff frequency $f_c \leq f_s/2$, sampled at rate f_s , and then quantized.

Figure 2: Orfanidis, p. 633, Figure 12.1.2

The prefilter ensures that the spectral images generated by the sampling process at integral multiples of f_s do not overlap, as required by the sampling theorem.

Figure 3: Orfanidis, p. 633, Figure 12.1.3

After digital processing, the sampled signal is reconstructed back to analog form by a D/A staircase reconstructor, followed by an analog anti-image lowpass postfilter with effective cutoff $f_s/2$.

Figure 4: Orfanidis, p. 634, Figure 12.1.4

The D/A converter, with its typical $\sin x/x$ response, removes the spectral images partially; the postfilter completes their removal. The combination of the staircase DAC and the postfilter emulates the ideal reconstructing analog filter.

The ideal reconstructor is a lowpass filter with cutoff the Nyquist frequency $f_s/2$. It has a very sharp transition between its passband, that is, the Nyquist interval, and its stopband.

In applications such as digital audio, to maintain high quality in the resulting reconstructed analog signal, a very high quality analog postfilter is required, which may be expensive. One way to alleviate the need for a high quality postfilter is to increase the sampling rate. This would cause the spectral images to be more widely separated and, therefore, require a less stringent, simpler, lowpass postfilter. This is depicted below for a new sampling rate that is four times higher than required, $f'_s = 4f_s$.

The *passband* of the postfilter extends up to $f_{\text{pass}} = f'_s/8 = f_s/2$, but its stopband need only begin at

Figure 5: Orfanidis, p. 634, Figure 12.1.5

Figure 6: Orfanidis, p. 634, Figure 12.1.6

$f_{\text{stop}} = f'_s - f'_s/8 = 7f'_s/8$. It is this wide transition region between passband and stopband that allows the use of a less stringent postfilter. For example, in oversampled digital audio applications, simple third-order Butterworth or Bessel analog postfilters are used.

The above approach, however, is impractical because it requires the actual resampling of the analog signal at the higher rate f'_s . For example, in a CD player the low rate samples are already stored on the CD at the prescribed rate of 44.1 kHz and the audio signal cannot be resampled.

3.2 Oversampled D/A

The philosophy of oversampling is to increase the sampling rate digitally using an interpolation filter which operates only on the available low-rate input samples. With respect to the new rate f'_s and new Nyquist interval $[-f'_s/2, f'_s/2]$, the spectrum of the low-rate samples depicted in Fig. 12.1.3 will be as shown in Fig. 12.1.8. This is also the spectrum of the high-rate upsampled signal at the output of the rate expander in Fig. 12.1.1.

Figure 7: Orfanidis, p. 635, Figure 12.1.8

A digital lowpass FIR filter with cutoff frequency $f_s/8$ and operating at the high rate f'_s , would eliminate the three spectral replicas that lie between replicas at multiples of f'_s , resulting in a spectrum that is identical to that of a signal sampled at the high rate f'_s , like that shown in Fig. 12.1.6.

The digital filter, being periodic in f with period f'_s , cannot of course remove the spectral replicas that are centered at integral multiples of f'_s . Those are removed later by the D/A reconstructor and the anti-image

analog postfilter.

The effect of such a digital filter on the spectrum of the low-rate samples is shown in Fig. 12.1.9, both with respect to the physical frequency f in Hz and the corresponding digital frequency, $\omega' = 2\pi f/f'_s$, in radians/sample.

Figure 8: Orfanidis, p. 636, Figure 12.1.9

In summary, a substantial part of the analog reconstruction process can be accomplished by DSP methods, that is, using a digital oversampling filter to remove several adjacent spectral replicas and thereby easing the requirements of the analog postfilter. The required sharp transition characteristics of the overall reconstructor are provided by the digital filter. Thus, the high-quality analog postfilter is traded off for a high-quality digital filter operating at a higher sampling rate. The overall system is depicted in Fig. 12.1.10.

Figure 9: Orfanidis, p. 636, Figure 12.1.10