

# 1 Lecture Outline

## Reading: Chapter 11 IIR Digital Filter Design

This lecture will cover the following topics

- Analog Chebyshev Lowpass Filter (Section 11.6.1)
- Digital Chebyshev Lowpass Filter (Section 11.6.6)

## 2 Analog Chebyshev Lowpass Filter (Section 11.6.6)

Chebyshev filters come in two varieties:

**Type 1** has equiripple passband and monotonic stopband

**Type 2** has equiripple stopband and monotonic passband

A typical Chebyshev magnitude response is shown in Fig. 11.6.10. For the same set of specifications, Chebyshev filters provide steeper transition widths and lead to smaller filter orders than the Butterworth filters. It is the equiripple property that is responsible for the narrower transition widths of these filters.

Figure 1: Orfanidis, p. 615, Figure 11.1.10

The magnitude-squared response of an  $N$ th order, Type 1 Chebyshev filter is given by

$$|H_a(\Omega)|^2 = \frac{1}{1 + \varepsilon_{\text{pass}}^2 C_N^2\left(\frac{\Omega}{\Omega_{\text{pass}}}\right)} \quad (1)$$

where  $C_N(X)$  is the Chebyshev polynomial of degree  $N$ , defined by

$$C_N(x) = \begin{cases} \cos(N \cos^{-1}(x)), & |x| \leq 1 \\ \cosh(N \cosh^{-1}(x)), & |x| > 1. \end{cases} \quad (2)$$

Within the passband range  $0 \leq \Omega \leq \Omega_{\text{pass}}$ ,  $C_N$  oscillates resulting in passband ripples. At the edge of the passband,  $|H_a(\Omega_{\text{pass}})|^2 = 1/(1 + \varepsilon_{\text{pass}}^2)$  while at  $\Omega = 0$  we have

$$|H_a(0)|^2 = \begin{cases} 1, & N \text{ odd} \\ \frac{1}{1 + \varepsilon_{\text{pass}}^2}, & N \text{ even.} \end{cases} \quad (3)$$

The transfer function  $H_a(s)$  of the Chebyshev filter can be constructed by determining the left-hand-plane poles of  $|H_a(\Omega)|^2$  and pairing them in conjugate pairs to form second-order sections. In this case we have,

$$H_a(s) = H_{a,0}(s) \prod_{i=1}^K H_{a,i}(s) \quad (4)$$

where

$$H_{a,i}(s) = \frac{\Omega_0^2 + \Omega_i^2}{s^2 - (2\Omega_0^2 \cos \theta_i) s + (\Omega_0^2 + \Omega_i^2)}, \quad (5)$$

$$H_{a,0}(s) = \begin{cases} \sqrt{\frac{1}{1 + \varepsilon^2}} & N = 2K, N \text{ even} \\ \frac{\Omega_0}{s + \Omega_0} & N = 2K + 1, N \text{ odd,} \end{cases} \quad (6)$$

$$\theta_i = \frac{\pi}{2N}(N - 1 + 2i), \quad (7)$$

$$\Omega_0 = \frac{\Omega_c}{\cosh \left[ \frac{1}{N} \cosh^{-1} (1/\varepsilon) \right]} \sinh(\mu), \quad (8)$$

$$\Omega_i = \frac{\Omega_c}{\cosh \left[ \frac{1}{N} \cosh^{-1} (1/\varepsilon) \right]} \sin(\theta_i), \quad (9)$$

$$\mu = \frac{1}{N} \ln \left( \frac{1}{\varepsilon} + \sqrt{\frac{1}{\varepsilon^2} + 1} \right), \quad (10)$$

and  $\Omega_c$  is the 3-dB down frequency. For the equation relating  $\Omega_c$  to  $\Omega_{\text{pass}}$ , see text (11.6.52).

### 3 Digital Chebyshev Lowpass Filter (Section 11.6.6)

The Chebyshev digital LPF formulas are arrived at using  $H_a(s)$  for the analog Chebyshev LPF and the bilinear transform.

$$H(z) = H_0(z) \prod_{i=1}^K H_i(z) \quad (11)$$

where

$$H_0(z) = \begin{cases} \sqrt{\frac{1}{1 + \varepsilon^2}} & , N = 2K (N \text{ even}) \\ \frac{G_0(1 + z^{-1})}{1 + a_{01}z^{-1}} & , N = 2K + 1 (N \text{ odd}) \end{cases} \quad (12)$$

and

$$H_i(z) = \frac{G_i(1 + z^{-1})^2}{1 + a_{i1}z^{-1} + a_{i2}z^{-2}}, \quad (13)$$

with

$$G_i = \frac{\Omega_0^2 + \Omega_i^2}{1 - 2\Omega_0 \cos \theta_i + \Omega_0^2 + \Omega_i^2}, \quad (14)$$

$$G_0 = \frac{\Omega_0}{\Omega_0 + 1}, \quad (15)$$

$$a_{i1} = \frac{2(\Omega_0^2 + \Omega_i^2 - 1)}{1 - 2\Omega_0 \cos \theta_i + \Omega_c^2 + \Omega_i^2}, \quad (16)$$

$$a_{i2} = \frac{1 + 2\Omega_0 \cos \theta_i + \Omega_0^2 + \Omega_i^2}{1 - 2\Omega_0 \cos \theta_i + \Omega_c^2 + \Omega_i^2}, \quad (17)$$

$$a_{01} = \frac{\Omega_0 - 1}{\Omega_0 + 1}, \quad (18)$$

$$\theta_i = \frac{\pi}{2N} (N - 1 + 2i), \quad (19)$$

$$\Omega_0 = \frac{\Omega_c}{\cosh \left[ \frac{1}{N} \cosh^{-1} (1/\varepsilon) \right]} \sinh(\mu), \quad (20)$$

$$\Omega_i = \frac{\Omega_c}{\cosh \left[ \frac{1}{N} \cosh^{-1} (1/\varepsilon) \right]} \sin(\theta_i), \quad (21)$$

and

$$\mu = \frac{1}{N} \ln \left( \frac{1}{\varepsilon} + \sqrt{\frac{1}{\varepsilon^2} + 1} \right). \quad (22)$$

The cutoff frequency has been prewarped according to  $\Omega_c = \tan(\omega_c/2)$  where  $\omega_c$  is the desired 3-dB down digital frequency.

In the EE395 DSP Toolkit (and in MATLAB's Signal Processing Toolbox), there is an implementation of the above digital Chebyshev LPF design formulas. Its use is as follows

```
[b,a] = chebyshv(N,epsilon,wc);
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