1 Lecture Outline

Reading: Chapter 6 Transfer Function

- Pole/Zero Designs: Comb Filters

2 Pole/Zero Designs: Comb Filters

If we again consider the generalized 2nd order resonator with $r = 1$ (notch filter) we have

$$\begin{aligned} H(z) &= \frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \\ &= \frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 + Rb_1 z^{-1} + R^2 b_2 z^{-2}} \\ &= \frac{N(z)}{N(R^{-1}z)} \end{aligned} \tag{1}$$

where

$$\begin{aligned} b_1 &= -2\cos(\omega_0) \\ b_2 &= 1 \\ a_1 &= -2R\cos(\omega_0) = Rb_1 \\ a_2 &= R^2 = R^2 b_2 \end{aligned} \tag{2}$$

and $N(z)$ has zeros at the notch locations

$$\begin{aligned} N(z) &= 1 + b_1 z^{-1} + b_2 z^{-2} \\ &= \left(1 - z_1 z^{-1}\right)\left(1 - z_1^* z^{-1}\right) \\ &= \left(1 - e^{j\omega_0} z^{-1}\right)\left(1 - e^{-j\omega_0} z^{-1}\right) \end{aligned} \tag{3}$$

The notch polynomial $N(z)$ can be generalized to include M notch frequencies

$$N(z) = \prod_{i=1}^{M} \left(1 - e^{j\omega_i} z^{-1}\right) \tag{4}$$

The denominator polynomial is chosen (for some $0 < \rho < 1$)

$$\begin{aligned} D(z) &= N(\rho^{-1}z) \\ &= \prod_{i=1}^{M} \left(1 - e^{j\omega_i} \rho z^{-1}\right) \end{aligned} \tag{5}$$

The zeros of $D(z)$ [poles of $H(z)$] lie in the same direction as the zeros of $N(z)$ [zeros of $H(z)$] but they are scaled by ρ and pushed inside the unit circle. So for each zero $z_i = e^{j\omega_i}$ there is a corresponding pole $p_i = \rho e^{j\omega_i}$.

The transfer function of the generalized notch filter is

$$\begin{aligned} H(z) &= \frac{N(z)}{N(R^{-1}z)} \\ &= \frac{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}} \\ &= \frac{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + \rho b_1 z^{-1} + \rho^2 b_2 z^{-2} + \cdots + \rho^M b_M z^{-M}} \end{aligned} \tag{6}$$

Figure 1: Orfanidis p. 252 Figure Frequency response of comb filter.

If ρ is near one, the distances from the movable point $e^{j\omega}$ to the pole/zero pairs are nearly the same except near the vicinity of the pair ($\omega = \omega_i$), where $|H(\omega)|$ is very small. These filters are generally used to cancel periodic interference such as power frequency pickup and its harmonics (multiples of the fundamental frequency).

Example 6.4.3

If we now move the unit circle zeros at the notch frequencies *behind* the corresponding pole,

$$z_i = e^{j\omega_i} \quad \rightarrow \quad z_i = re^{j\omega_i} \tag{7}$$

where $r < \rho$, we covert notch dips into sharp peaks. The transfer function in this case is

$$\begin{aligned} H(z) &= \frac{N(r^{-1}z)}{N(\rho^{-1}z)} \\ &= \frac{1 + rb_1z^{-1} + r^2b_2z^{-2} + \cdots + r^Mb_Mz^{-M}}{1 + \rho b_1z^{-1} + \rho^2b_2z^{-2} + \cdots + \rho^Mb_Mz^{-M}} \end{aligned} \tag{8}$$

Generically, notching and peaking filters are referred to as comb filters.

Figure 2: Orfanidis p. 254 Figure Frequency response of comb filter.