New Mexico State University
Klipsch School of Electrical Engineering

EE395 - Introduction to Digital Signal Processing

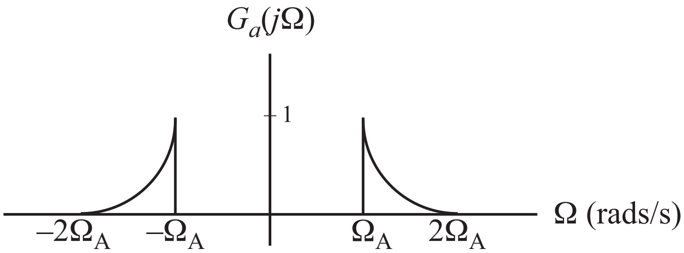
Fall 2009
Exam #2

Name: ____________________

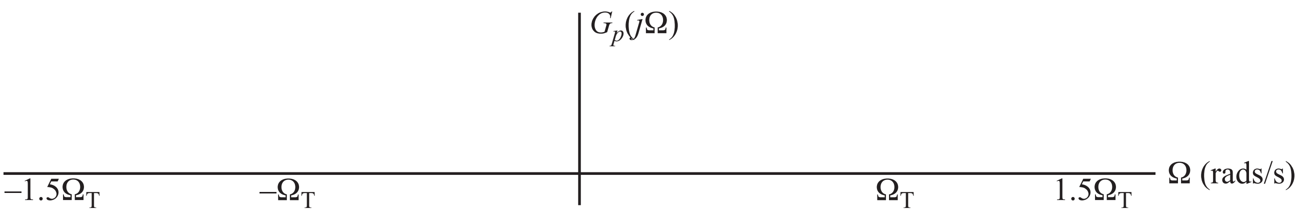
Prob. 1	/ 20 points
Prob. 2	/ 20 points
Prob. 3	/ 20 points
Prob. 4	/ 20 points
Prob. 5	/ 20 points
Bonus	/ 7 points
Total	/ 100 points

Prob. 1

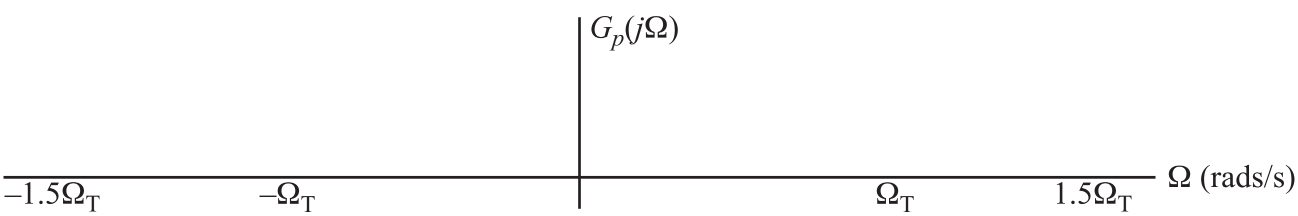
Let $g_a(t)$ be a continuous-time signal and denote its spectrum as $G_a(j\Omega)$ which is illustrated below. Consider the ideal sampling model where $g_a(t)$ is sampled at a rate $\Omega_T = \dfrac{2\pi}{T}$ where T is the sample period. For the given Ω_T, sketch the spectrum of the sampled signal, $G_p(j\Omega)$ for $-1.5\Omega_T \leq \Omega \leq 1.5\Omega_T$. Be sure to label important edge frequencies.



(a) $\Omega_T = 4\Omega_A$. Aliasing (any overlap in spectral replicas) is present (YES / NO)?

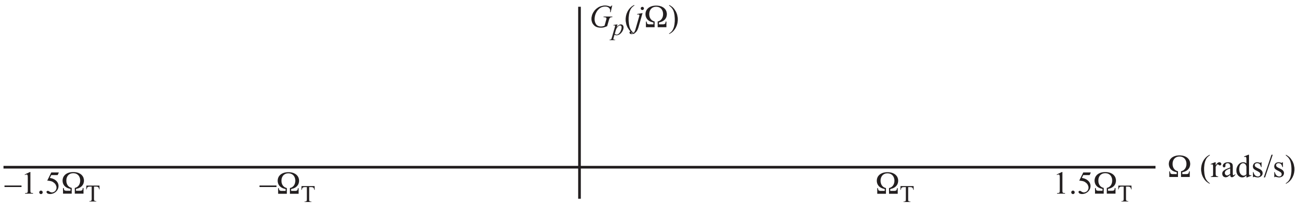


(b) $\Omega_T = 2\Omega_A$. Aliasing (any overlap in spectral replicas) is present (YES / NO)?



Prob. 1 (cont.)

(c) $\Omega_T = 1.5\Omega_A$. Aliasing (any overlap in spectral replicas) is present (YES / NO)?



Prob. 2

Determine a causal and stable transfer function $H(z)$ the meets the following requirements:

1. the filter coefficients are real-valued
2. for $f_s = 20$ kHz, the magnitude response at $f_0 = 5$ kHz is exactly 0.
3. the filter has a linear phase response
4. the system has a zero at $z = -1/2$
5. the frequency response of the system at 0 Hz is +1.0

You may add additional poles and zeros and scale the transfer function as needed in order to meet the requirements.

Prob. 3

Determine the z-transform and Region of Convergence (ROC) for the following.

(a) $x[n] = \sum_{k=-2}^{2} k\delta[n-k]$.

(b) $x[n] = -2\delta[n] + \frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$. Express $X(z)$ as a rational function.

Prob. 3 (cont.)

Determine the inverse z-transform for the following.

(c) $X(z) = \dfrac{1 - 2z^{-1}}{z^{-1} - 2}$ and ROC $|z| > \frac{1}{2}$

(d) $X(z) = \dfrac{1 - \frac{1}{3}z^{-1}}{(1 + \frac{1}{2}z^{-1})(1 - \frac{2}{3}z^{-1})}$ and $x[n]$ has both left and right sides.

Prob. 4

A *causal* LTI DT system is described by the transfer function

$$H(z) = \frac{1 - 0.5z^{-1}}{(1 - 0.8z^{-1})(1 + 0.8z^{-1})}.$$

(a) Sketch the pole-zero plot and determine the ROC.

Determine the following equivalent descriptions.

(b) Linear, constant-coefficient difference equation (LCCDE), $\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$

Prob. 4 (cont.)

(c) Impulse response, $h[n]$.

(d) Based on the pole-zero plot, sketch the magnitude response $|H(e^{j\omega})|$ vs. ω for $0 \leq \omega \leq \pi$. Be sure to note the values of $|H(e^{j0})|$ and $|H(e^{j\pi})|$ on your graph.

Prob. 5

Consider two practical D/A converters, zero-order hold and linear interpolators, whose impulse and frequency responses are given by

$$h_z(t) = \begin{cases} 1, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases} \quad \leftrightarrow \quad H_z(j\Omega) = e^{-j\Omega T/2} \left[\frac{\sin(\Omega T/2)}{\Omega/2} \right]$$

and

$$h_{LI}(t) = \begin{cases} 1 - \dfrac{|t-T|}{T}, & 0 \le t < 2T \\ 0, & \text{otherwise} \end{cases} \quad \leftrightarrow \quad H_{LI}(j\Omega) = e^{-j\Omega T} \left[\frac{\sin(\Omega T/2)}{\Omega/2} \right]^2.$$

where T is the sampling period.

(a) Sketch $h_{LI}(t)$ vs. t and $|H_{LI}(j\Omega)|$ vs. Ω for $0 \le \Omega \le 3\Omega_T$ where $\Omega_T = 2\pi/T$. The side lobes in $|H_{LI}(j\Omega)|$ are (LARGER / SMALLER) than $|H_z(j\Omega)|$ (circle one).

(b) For $y_p(t)$ given on p. 222 Fig. 4.53(a), sketch $y_{LI}(t) = h_{LI}(t) * y_p(t)$ for $-2T \le t \le 3T$.

Prob. 5 (cont.)

(c) For $|Y_p(j\Omega)|$ given on p. 223 Fig. 4.55(b), sketch $|Y_{LI}(j\Omega)| = |H_{LI}(j\Omega)Y_p(j\Omega)|$ for $0 \leq \Omega \leq 3\Omega_T$. The spectral replicas in $|Y_{LI}(j\Omega)|$ are (LARGER / SMALLER) than $|Y_Z(j\Omega)|$ (circle one).

Bonus Prob (+7 points)

(a) Show that $h_{LI}(t) = h_Z(t) * h_Z(t)$ from Problem 5. Then show how one can easily derive $H_{LI}(j\Omega)$ from $H_z(j\Omega)$.

(b) We would expect that a linear interpolator would be "better" than a zero-order hold interpolator. Suppose we define "better" to mean better suppression of high-frequency spectral replicas. Use your sketch in Problem 5(c) to argue this point.