EE395: Introduction to Digital Signal Processing

Midterm #2

November 7, 2008

You are allowed to use a dumb calculator on this test and one 8.5x11" notesheet (both sides may be written on). You are not allowed to use the textbook, homework solutions, or any other references. Your answers must be written in the space provided on the exam sheets, but you may attach additional sheets containing your work if necessary. Do not talk during the test: if you have questions, ask the exam proctor. Show your work (including intermediate steps) unless otherwise notes in the problem. You may use properties but you **must** state which property you are using when you use it.

Name:____

Problem Number	***Max Points***	***Points***
1	30	
2	30	
3	10	
4	30	
Total	100	

1. (30 pts) Let $h(n) = (2)^{n-2}\, u(-n-1) + (1/4)^{n}\, u(n+1)$ be the impulse response of a system.

a) (10 pts) Find the transfer function $H(z)$ for this system along with its associated region of convergence (ROC). You do not need to put answer over common denominator.

b) (5 pts) Is the system stable? Briefly justify your answer.

c) (5 pts) Is the system causal? Briefly justify your answer.

d) (10 pts) List all of the ROCs that are *possible* for the $H(z)$ found in part a)? What kind of sequence--left-sided, right-sided, two-sided, or finite length-- is associated with each of these ROCs? Note that only one of these possible ROCs corresponds to the actual impulse response given in the problem statement.

2. (30 pts) Let the output of a system *Y(z)* be given by

$$Y(z) = \frac{1+3z^{-1}}{(1-2z^{-1})(1+0.75z^{-1})}$$

with ROC $|z| > 2$ and let the input sequence be $x[n] = (2)^n \mu[n-1] + 3\,\delta[n]$.

a) (10 pts) Find *X(z)* and its ROC. Write as rational polynomial in z^{-1} for full credit (i.e., put it over a common denominator).

b) (10 pts) Find the transfer function *H(z)* and the associated ROC for this system. *Plot the poles and zeros as well*. Hint: Recall the convolution property of *z*-transform.

b) (10 pts) Find the impulse response *h(n)*. Is this system stable. Justify briefly.

3. (10 pts) True or False

a) A time sequence can have a z-transform even if its DTFT does not exist:_________

b) Since the DFT can be found by sampling the DTFT, if the DFT exists, then the DTFT corresponding to the original time sequence must also exist:________

c) If the DFT is conjugate symmetric, then the corresponding time domain sequence must always be real:_______

d) It is always possible to use DFTs to implement linear convolution if at least one of the sequences being convolved has finite length:________

e) From only the region of convergence (ROC) of its z-transform, we can determine whether or not a sequence is finite length:________

f) If the poles of its z-transform are all outside of the unit circle, then there is no way that the corresponding sequence can be absolutely summable:________

g) Overlap-save linear convolution has lower computational complexity than overlap-add convolution:_______

h) The conjugate symmetry and anti-symmetry properties are what make it possible to compute a $2N$-point DFT for a real sequence with the complexity of only an N-point DFT:_______

i) The type of discrete cosine transform (DCT) we get is determined by what type of periodic symmetric extension we apply:________

j) Type 1 and type 3 sequences are always symmetric about the midpoint of their finite length intervals while type 2 and type 4 sequences are anti-symmetric:________

4. (30 pts) Even *more* fun with DFTs!

a) (7 pts) The following 4 samples of a length-8 sequence $x[n]$ whose DFT $X[k]$ is entirely imaginary are given as follows: $x[1] = j + 1$, $x[2] = 5j$, $x[3] = -2$, $x[4] = -j - 1$. We are also told that $x[0]$ is real. Find the remaining samples of the sequence.

b) (9 pts) Given $x[n]$ in part a), if $G[k] = (-j)^k X[k]$, find the corresponding sequence $g[n]$ without explicitly evaluating any transforms or inverse transforms. If you could not find an answer in part a), simply assume for the purposes of this problem that the remaining samples of $x[n]$ are all zero.

c) (7 pts) Let $X[k]$, $0 \le k \le 9$ be the 10-point DFT of an entirely imaginary sequence $x[n]$ with the first 6 samples of $X[k]$ being given by: $X[k] = \{0, 1 + j, 2 - 3j, j, -3, 0\}$, $0 \le k \le 5$. Determine the remaining 4 DFT coefficients.

d) (7 pts) We wish to apply overlap-add convolution to an infinite-length input sequence $x[n]$ using a filter with impulse response $h[n]$. Because of system considerations, it is most efficient to divide the input into non-overlapping blocks of length 25. Also, we wish to implement the required linear block convolutions using a 64-pt DFT chip which is readily available. What is the longest impulse response of filter $h[n]$ that we can use here? Justify your answer