EE395: Introduction to Digital Signal Processing

Midterm #1

October 3, 2008

You are allowed to use a dumb calculator (of the type specified in the syllabus) on this test and *one* 8.5x11" notesheet (both sides may be written on). You are not allowed to use the textbook, homework solutions, or any other references. Your answers must be written in the space provided on the exam sheets, but you may attach additional sheets containing your work if necessary. Do not talk during the test: if you have questions, ask the exam proctor. Show your work (including intermediate steps) unless otherwise notes in the problem. You may use properties but you **must** state which property you are using when you use it.

Name:_____

Problem Number	***Max Points***	***Points***
1	20	
2	25	
3	30	
4	25	
Total	100	

1. (20 pts) True or False

a) A causal and stable LTI system must have a left-sided impulse response:_____

b) Sampling an analog signal above the Nyquist rate will result in aliasing:_____

c) If a system cannot be described by an impulse response then it is not stable:_____

d) Shifting a sequence in the time domain results in a modulation of the original sequence's DTFT:_____

e) If a sequence is two-sided, it cannot have a DTFT because the sum will not converge:_____

f) A periodic analog signal that is sampled will always result in a periodic discrete-time sequence:_____

g) The sequence $x[n] = n$, for n going from 0 to infinity is bounded:_____

h) The impulse response corresponding to a linear constant coefficient difference equation (LCCDE) will generally be infinite in length:_____

i) It is possible to use an LCCDE to implement a system having a two-sided impulse response:_____

j) The DTFT of a sequence will be real only if that sequence is also real:_____

2. (25 pts) Discrete-time Fourier Transform (DTFT). Clearly state which properties (if any) you have used and show your work.

a) (10 pts) Determine the DTFT of the following sequence:
$x[n] = \alpha^{n+1}\mu[n-1] + \beta^{n-1}\delta[-n+2]$. ***Define the range of α and β for which this DTFT will exist.***

b) (10 pts) Determine the inverse DTFT *h[n]* of $H(e^{j\omega}) = \dfrac{3e^{-j4\omega}}{1-2e^{j\omega}}$. Use properties.

c) (5 pts) Let the *h[n]* found in part b) correspond to the impulse response of a system. Is *h[n]* right, left, or two-sided? Is *h[n]* absolutely summable? Justify your answers briefly.

3. (30 pts) Short Answer

a) (5 pts) Given a system with impulse response $h[n] = (¼)^n \mu[n]$. If the input sequence $x[n]$ has finite length, can the output sequence $y[n] = h[n] * x[n]$ be calculated on an actual computer (i.e., progressively outputting $y[n]$ for increasing n) directly using the convolutional sum equation directly? Explain why or why not.

b) (5 pts) What is an IIR discrete-time system? Define. How can we implement an IIR system on an infinite-length input sequence in practice (i.e., on a computer)? Justify your answer briefly.

c) (5 pts) What is the difference between a recursive and a non-recursive system? Is it possible to implement IIR systems in both recursive and non-recursive forms? Explain your reasoning, giving examples if need be.

d) (5 pts) What is an FIR discrete-time system? Define. What is the only way for an FIR system to be unstable (in a BIBO sense)?

e) (5 pts) Is the following sequence (you do not need to show your work if answer is correct)

- Causal, anti-causal, noncausal, or none of these?
- Periodic, aperiodic, or neither?
- Conjugate symmetric, conjugate antisymmetric, or neither?
- Finite length, infinite length, both, or neither?

$\{x[n]\} = \{2j \quad -3j \quad \underset{\uparrow}{-j} \quad -3j \quad 2j\}$

f) (5 pts) For the discrete-time systems described below, determine whether or not it is (1) linear, (2) causal, (3) stable, and (4) shift/time invariant. No work needed.

$$y[n] = \begin{cases} \cos(x[n]), & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

4. (25 pts) You have been asked to design a real-time digital filtering system that eliminates a band of frequencies between 25 and 30 MHz but preserves everything else. The system uses an analog-to-digital converter (ADC) to digitize the analog input, a digital signal processing chip (DSP) to filter the digitized signal, and a digital-to-analog (DAC) converter to convert the signal back to analog. Assume that ideal sampling is used as well as ideal anti-aliasing and reconstruction filters. The maximum frequency that needs to be preserved in the reconstructed analog signal is 65 MHz.

a) (5 pts) What is the lowest sampling rate allowable in this system given the above constraints?

b) (10 pts) Draw plots of the ideal anti-aliasing and reconstruction filters for this system assuming the sampling rate from part a).

c) (10 pts) Assume now that the ADC has a sampling rate of 150 MHz and that the digital filter is ideal. Plot the magnitude of the frequency response of the digital filter, carefully labeling all of the frequency transition points. Show your work in calculating these frequency transition points.