EE395: Introduction to Digital Signal Processing

Final

December 10, 2007

You are allowed to use a dumb calculator on this test and two 8.5x11" notesheet (both sides may be written on). You are not allowed to use the textbook, homework solutions, or any other references. Your answers must be written in the space provided on the exam sheets, but you may attach additional sheets containing your work if necessary. Do not talk during the test: if you have questions, ask the exam proctor. Show your work (including intermediate steps) unless otherwise notes in the problem. You may use properties but you **must** state which property you are using when you use it.

Name:____

Problem Number	***Max Points***	***Points***
1	20	
2	20	
3	20	
4	20	
5	20	
Bonus	10	
Total	100	

1. (20 pts) Given the linear constant coefficient difference equation (LCCDE)

$$y[n] + 0.75y[n-1] - 0.625y[n-2] = 4x[n-1]$$

and that

$x[n] = \beta\,\mu[n]$ with initial conditions $y[-1] = 1$ and $y[-2] = 0$.

Find a closed-form solution for $y[n]$. Just to remind you, such a solution will not depend upon x[n] nor will it contain any summations.

2. (20 pts) *Z*-transform problems

a) (10 pts) Let $x[n] = 2\delta[n-1] + (-0.7)^{n-1}\,\mu[n+1]$ and $h[n] = (1.2)^{n+1}\,\mu[n]$. Find $X(z)$ and $H(z)$ along with their ROCs. What would be the ROC for $Y(z) = H(z)X(z)$? Does the DTFT of $y[n]$ exist? Explain.

b) (10 pts) Let the transfer function *H(z)* and the input *X(z)* be as follows:

$$H(z)=3z^{-1}+\frac{2z^{-1}}{1+1.4z^{-1}}$$, ROC: |z| < *1.4*

and

$$X(z)=\frac{1}{1-0.6z^{-1}}$$, ROC: |z| < *0.6*

Find the output *y[n]* where *Y(z)* = *ZT{y[n]}* = *H(z)X(z)*. What is the ROC of *Y(z)*? Is y[n] absolutely summable? Justify you answer.

3. (20 pts) You have been asked to design a real-time digital filtering system that eliminates a band of frequencies between 50 and 60 MHz but preserves everything else. The system uses an analog-to-digital converter (ADC) to digitize the analog input, a digital signal processing chip (DSP) to filter the digitized signal, and a digital-to-analog (DAC) converter to convert the signal back to analog. Your boss would like the maximum frequency output by the system's DAC to be as high as possible given the various technological constraints.

Constraints:

- the highest sampling rate ADC/DAC available operate at a rate of 1 GHz (any rate below this is also available, of course),
- for the digital bandstop filter (used to eliminate the 50-60 MHz frequency band) to have adequate passband ripple, stopband attenuation, and transition bandwidths, a length 10 minimum-phase FIR filter is used,
- The fastest DSP available on which to run the FIR bandstop filter can perform 1 multiplication and 1 addition in a single instruction cycle and it executes 2 billion instructions per second. Assume that no overhead for loops or other control structures is required in the implementation of the FIR filter.

a) (10 pts) What is the highest frequency that can be retained at the output of the proposed system given the technology constraints listed above? What is the factor limiting this maximum frequency? Explain briefly.

b) (10 pts) Assuming that the ADC samples the signal at $\Omega_T = 400\pi$ M radians/second, what are the two positive cutoff frequencies ω_{c1} and ω_{c2} for the digital bandstop filter in radians/sample? You can assume here that the transition bands have zero width. Show your work.

4. (20 pts) Filter implementation & Design

Given the transfer function *H(z)* for a system that is formed as the cascade of systems *G(z), V(z)*, and *W(z)*, i.e., *H(z) = G(z)V(z)W(z)*, where

$$G(z)=\frac{3+z^{-1}}{1+0.1z^{-1}} \quad , \quad V(z)=\frac{2+3z^{-1}}{2-z^{-1}} \quad , \quad W(z)=\frac{z^{-1}}{1-1.2z^{-1}}$$

a) (10 pts) Draw the block diagram for the direct form II implementation of *H(z)*. Show your work and clearly give the values of all of the multipliers in the system. Is the resulting system cannonic? Briefly justify your answer

b) (10 pts) Assuming that the filter *W(z)* above was designed using the bilinear transform method with $T = 2$ from an analog (continuous-time) filter *W(s)*. Find *W(s)* as a rational function of polynomials in *s*. Show your work.

5. (20 pts) Given an LTI system with impulse response $h[n] = (-0.8)^n \mu[n] - (-0.8)^n \mu[n-5]$.

a) (10 pts) Determine the frequency response $H(e^{j\omega})$ of this system. Your solution should contain no summations.

b) (10 pts) Let $H[k] = H(e^{j\omega})|_{\omega=\frac{2\pi k}{N}}$. In other words, $H[k]$ is the frequency response of the system from part a) sampled at N uniformly spaced points. For what value of N will $h[n] = DFT^{-1}\{H[k]\}$ where DFT^{-1} is the inverse discrete Fourier transform? Briefly explain your reasoning.

Bonus Problem (10 pts extra credit): *Do NOT use more than the space provided.*

a) (5 pts) Explain how a spectrogram is computed.

b) (5 pts) Explain how adaptive noise cancellation works.