

EE395: Introduction to Digital Signal Processing
Midterm #2
October 17, 2007

You are allowed to use a dumb calculator on this test and one 8.5x11" notesheet (both sides may be written on). You are not allowed to use the textbook, homework solutions, or any other references. Your answers must be written in the space provided on the exam sheets, but you may attach additional sheets containing your work if necessary. Do not talk during the test: if you have questions, ask the exam proctor. Show your work (including intermediate steps) unless otherwise notes in the problem. You may use properties but you **must** state which property you are using when you use it.

Name: _____

<i>Problem Number</i>	<i>Max Points</i>	<i>Points</i>
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 pts) Given an LTI system with impulse response $h[n] = (-0.7)^n \mu[n-1]$. **Show your work.**
- a) (10 pts) Determine the frequency response $H(e^{j\omega})$ of this system and evaluate its value at $\omega = \pm\pi/4$.
- b) (10 pts) What is the steady-state output $y[n]$ of the system for input $x[n] = \cos(\pi n/4)\mu[n]$?

2. (20 pts) It is your first day on the job and you have been asked to figure out what an ancient digital signal processing system is doing, so that you can someday (but *not* today) redesign it with modern hardware. By studying the available documentation, you have determined the following about the system:

- Sampling Period: 3 microseconds
- The input to the sampler is bandlimited to the Nyquist frequency.
- Quantizer Resolution: 8 bits/sample
- Digital filter has 2 notches (narrow regions where frequencies are blocked): at $\pm 0.1\pi$ and at $\pm 0.45\pi$ radians/sample.

Clearly justify answers or show your work.

a) (5 pts) How many bits per second of data must be processed by this system?

b) (5 pts) What is the highest analog input frequency in radians/second that can be represented in the output of the digital filter?

c) (10 pts) In terms of analog frequency (in Hz), which analog frequencies are eliminated by the digital filter. In other words, if we converted the output of the digital filter back into an analog signal, what frequency components would be missing. **Show your work.**

3. (20 pts) True or False

- a) If we uniformly sample the DTFT to create the DFT, the sequences resulting from the inverses of each of these transforms (i.e., the inverse of the DTFT and the inverse of the DFT created by sampling the DTFT) will always be identical: _____
- b) The DFT will always exist when applied to any finite-valued real or complex input sequence: _____
- c) If the input sequence is conjugate symmetric, then the resulting DFT must always be real: _____
- d) It is impossible to use the DFT to implement linear convolution: _____
- e) Aliasing results whenever we sample a signal at a frequency that is more than 2 times higher than the highest frequency in our input: _____
- f) Once aliasing is introduced into the sampled (discrete-time) signal, that entire signal is corrupted (i.e., it contains no usable information): _____
- g) Overlap-Save linear convolution uses circular convolution on overlapping blocks of the input sequence: _____
- h) Overlap-Add linear convolution cannot be implemented using a DFT because it performs linear convolution on non-overlapping blocks of the input signal: _____
- i) The discrete cosine transform (DCT) is popular in image and audio compression because it compacts energy less than either the DFT or the Haar transformations: _____
- j) If the length of an input sequence is odd, then that sequence cannot have either Type 3 or Type 4 (anti-) symmetry: _____

4. (20 pts) Fun with DFTs!

a) (5 pts) The first 6 samples of a length-10 real sequence $x[n]$ whose DFT $X[k]$ is real are given as follows: $x[0] = 2$, $x[1] = 1$, $x[2] = 5$, $x[3] = -4$, $x[4] = 7$, $x[5] = 3$. Find the remaining samples of the sequence.

b) (5 pts) Given $x[n]$ in part a), if $G[k] = (-1)^k X[k]$, find the corresponding sequence $g[n]$ without explicitly evaluating any transforms or inverse transforms. If you could not find an answer in part a), simply assume for the purposes of this problem that the remaining samples of $x[n]$ are all zero.

c) (5 pts) Let $X[k]$, $0 \leq k \leq 9$ be the 10-point DFT of a real sequence $x[n]$ with the first 6 samples of $X[k]$ being given by: $X[k] = \{5, 1 + j, 2 - 3j, j, -3, 0\}$, $0 \leq k \leq 5$. Determine the remaining 4 DFT coefficients.

d) (5 pts) For $X[k]$ as found in part c), determine $x[0]$ without explicitly evaluating the inverse transformation of $X[k]$.

5. (20 pts) Given a length-6 sequence $x[n]$ with associated DFT $X[k]$, a length-6 sequence $g[n]$ with associated DFT $G[k]$, and a length-6 sequence $v[n]$ with associated DFT $V[k]$.

a) (10 pts) Let $G[k] = X_{CS}[k]$ and $V[k] = X_{CA}[k]$. Given that $g[n] = \{1, 2, 3, 4, 5, 6\}$ and $v[n] = \{6j, 5j, 4j, 3j, 2j, j\}$, find $x[n]$. Note: you do not need to evaluate any DFTs or IDFTs to accomplish this.

b) (10 pts) Alternately, let $G[k] = \text{real}\{X[k]\}$ and $V[k] = \text{imaginary}\{X[k]\}$ and let $x[n] = \{1, 2, 3, 4, 5, 6\}$. Find $g[n]$ and $v[n]$. Again, no DFTs or IDFTs are required.