New Mexico State University
Klipsch School of Electrical Engineering

EE555 - Advanced Linear Systems
Spring 2005
Exam #1

Name: ____________________

Prob. 1		/ 5 points
Prob. 2		/ 5 points
Prob. 3		/ 10 points
Prob. 4		/ 15 points
Prob. 5		/ 25 points
Prob. 6		/ 25 points
Prob. 7		/ 15 points
Total		/ 100 points

Prob. 1

Show that if $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices, then $\mathbf{AB}$ and $\mathbf{BA}$ have the same characteristic polynomial or equivalently have the same eigenvalues even if $\mathbf{AB} \neq \mathbf{BA}$.

Prob. 2

Find the minimum polynomial, $\mu(s)$ of the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

Prob. 3

(Mullis p. 54 Exercise) With

$$\mathbf{N}_i \quad = \quad \mathbf{E}_i \left(\mathbf{A} - \lambda_i \mathbf{I} \right)$$

show that

(i) $\mathbf{N}_i^k = \mathbf{E}_i \left(\mathbf{A} - \lambda_i \mathbf{I} \right)^k$

(ii) $\mathbf{N}_i^{n_i} = \mathbf{0}$ where n_i is the multiplicity of λ_i

(iii) For $i \neq j$, $\mathbf{N}_i \mathbf{E}_j = \mathbf{E}_j \mathbf{N}_i = \mathbf{N}_i \mathbf{N}_j = \mathbf{0}$

Prob. 4

Let

$$\mathbf{b} \;=\; \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

and consider the system of linear equations

$$\mathbf{A}\mathbf{x} \;=\; \mathbf{b}$$

For each of the following matrices, determine the subspaces Range($\mathbf{A}$) and Null($\mathbf{A}$). Based on these subspaces also determine whether or not there is a solution for $\mathbf{x}$ and if so, whether it is unique or not. You do not have to actually compute $\mathbf{x}$ although you may choose to do so if it helps.

(i) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

(ii) $\mathbf{A} = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$

(iii) $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(iv) $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

(v) $\mathbf{A} = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$

Prob. 5

Consider the discretized Newtonian equations for motion

$$\begin{aligned} d(n+1) &= d(n) + v(n)T + \frac{1}{2}a(n)T^2 \\ v(n+1) &= v(n) + a(n)T \end{aligned}$$

where $d(n)$, $v(n)$, and $a(n)$ are the distance, veleocity, and acceleration (respectively) at time index n and T is the sample period. Define a state vector,

$$\mathbf{x}(n) = \begin{bmatrix} d(n) \\ v(n) \\ a(n) \end{bmatrix}.$$

(i) Let

$$\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n)$$

be a *state update equation.* Determine the *state transition matrix*, $\mathbf{A}$ assuming constant acceleration, i.e. $a(n+1) = a(n)$.

(ii) Show that

$$\mathbf{x}(n+k) = \mathbf{A}^k\mathbf{x}(n).$$

(iii) For $\mathbf{A}$ found in (i), determine the eigenvalues, λ_i and multiplicities, n_i; the projections, $\mathbf{E}_i$; and (if applicable) the nilpotent matrices, $\mathbf{N}_i$. Do make make any assumptions on T.

(iv) Assuming an initial distance of 0 m, initial velocity of 0 m/s, constant acceleration of 1 m/s^2, and a sample period of 1 s, determine the distance and velocity at $n = 100$. You *must* use your results in (iii) and the function calculus [either or p.42 (12) or p. 54 (18)] to compute your answer—direct calculation using the Newtonian equations (with $T = 100$) or brute force matrix multiplications will not be accepted.

Prob. 6

Repeat Prob. 5, except that now the acceleration doubles every sample period, i.e.

$$a(n+1) \quad = \quad 2a(n)$$

(i) Determine the state transition matrix, $\mathbf{A}$.

(ii) For $\mathbf{A}$ found in (i), determine the eigenvalues, λ_i and multiplicities, n_i; the projections, $\mathbf{E}_i$; and (if applicable) the nilpotent matrices, $\mathbf{N}_i$. Do make make any assumptions on T.

(iii) Assuming an initial distance of 0 m, initial velocity of 0 m/s, initial acceleration of 1 m/s^2, and a sample period of 1 s, determine the distance, velocity, and acceleration at $n = 10$. You *must* use your results in (ii) and the function calculus [either or p.42 (12) or p. 54 (18)] to compute your answer—brute force matrix multiplications will not be accepted.

Prob. 7

(Mullis p. 41 Exercise) Let

$$\psi(s) = \begin{bmatrix} 1 \\ s \\ \vdots \\ s^{n-1} \end{bmatrix}$$

and

$$\begin{aligned} \mathbf{V} &= \left[\begin{array}{ccc} \psi(\lambda_1), & \ldots, & \psi(\lambda_n) \end{array} \right] \\ &= \text{Vandermonde}(\lambda_1, \ldots, \lambda_n) \end{aligned}$$

(assume distinct $\{\lambda_1, \ldots, \lambda_n\}$). Show that

(i)

$$\begin{bmatrix} e_1(s) \\ \vdots \\ e_n(s) \end{bmatrix} = \mathbf{V}^{-1}\psi(s)$$

Hint: Solve $\mathbf{V}\mathbf{x} = \psi(s)$ and show $x_k = e_k(s)$.

(ii)

$$\sum_{k=1}^{n} \lambda_k^m e_k(s) = s^m, \quad \text{for } 0 \le m \le n-1$$