Blind Equalization

These introductory remarks are based on

Sklar, *Digital Communications*, Prentice-Hall.

Introduction to Digital Communications

Digital modulation is the process by which digital symbols (groups of bits) are transformed into analog waveforms (using a transmitting filter) that are compatible with the characteristics of a channel, such as a telephone line, coaxial cable, or the atmosphere. Typically, the symbol stream modulates a sinusoid called a carrier.

Figure: Typical digital communication system (Sklar, p. 5)

The objective in digital communications is to design a system that accommodates the highest possible data rate, subject to a specified bit error rate (BER). However, distortion of the waveform is caused the combined effects of the transmit and receive filters as well as the channel. This distortion causes a spreading (due to the convolution of the waveform with the filters) of the individual waveforms and subsequent overlap. This distortion is more commonly called Intersymbol Interference (ISI).

In order to decode (demodulate) the received noisy and filtered waveform into its original binary data (detection), we employ a maximum likelihood (ML) detector implemented as a bank of correlators or matched filters. Matched filters are custom designed for each possible waveform corresponding to the data so that the filter output is positive when the input is the signature waveform and is negative when the input is any other waveform.

The matched filter output is sampled at the symbol rate and a decision (based on the sampled output) regarding the original transmitted symbol is made.

Supervised Adaptive Filtering: Conventional Equalization

The typical approach in coping with the channel distortion is to employ an adaptive equalizer to “unravel” the effects of the channel on the waveform. The basic setup is shown below.

Figure 9.19

The adaptive equalizer has the task of correcting for channel distortion in the presence of additive white noise. In order to train the adaptive filter, a replica (training sequence) of the desired response is stored in the receiver. Standard adaptive algorithms are then used to adjust the filter coefficients to minimize the MSE between the equalizer output and the training sequence.

Unsupervised Adaptive Filtering: Blind Equalization

These introductory remarks are based on

J. Treichler, C. R. Johnson, and M. Larimore, *Theory and Design of Adaptive Filters*, Prentice-Hall.

Introduction
In the model of adaptive filtering that we have assumed, a desired or reference signal is available. However, there are many problems encountered in signal processing where the availability of a reference signal is either not possible or impractical.

Classic Approaches to Coping with the Lack of a Reference Signal

We consider two classic approaches with how to deal with the lack of a reference signal.

1) Prearrangement of a Suitable Reference Signal

As we've seen in the adaptive equalization problem, we pre-arrange for a signal to be used at both the transmitter and receiver to "train" the equalizer located at the receiving end. During the training period, the receiver compares the equalizer's output with its own locally stored version of the prearranged signal. The error signal is then used in the adaptive algorithm to adjust the filter coefficients.

The technique is problematic in a number of ways. Among the most important is that it works only for a channel that in unknown initially but otherwise time-invariant. Once the equalizer has adapted and the transmitted switches over to sending the user's data, the equalizer's adaptation must stop. If the channel changes its characteristics, however, the equalizer is no longer appropriate and the quality of the receiver's output declines.

Under normal operating conditions (after the adaptive equalizer has been trained), a good facsimile of the transmitted sequence is being produced at the output of the decision device in the receiver. Accordingly, if this output were the correct transmitted sequence, it may be used as the "desired" response for the purpose of continuing adaptive equalization.

Figure 6.4: Decision Directed equalization

Such a method of learning is said to be decision-directed because the receiver attempts to learn by employing its own decisions. If the average error probability is small, the decisions made by the receiver are correct enough for the estimates of the error signal to be accurate most of the time. This means that the equalizer is able to improve the adaptive filter coefficients which will in turn lower the average probability of symbol error.

However, it is possible for the reverse effect to occur, in which case the equalizer will lose acquisition of the channel.

2) Using a Function of the Input Signal Itself as the Reference

One obvious approach is to use some function of the input signal itself as the reference waveform. As a simple example we consider the adaptive (spectral) line enhancer (ALE).

Figure 6.3: ALE

In this application, we would like to extract or enhance a tone (sinusoid or other narrowband phenomena) from a broadband noise signal. With this setup, the filter output is the tone of interest. Under the assumption that the broadband signal is white, the delay, Δ acts to decorrelate the broadband noise portions of "reference signal" with that of the input to the filter. Due to the wave nature of the tone, any delayed version of tone will always be correlated to itself.

The Property Restoration Concept

We should note that the above approaches, while useful and successful, do not really meet the original objective of operating without a reference signal. In both cases, the input signal is used to fashion a proxy for the reference signal. We now consider the original objective of unsupervised adaptive filtering.

Property-restoration approach: exploit the fact that many signals, particularly digital communications signals, have certain invariant properties that can be sensed and then used as the basis for adapting a filter.

Example: Many communications signals such as FM and PM commonly employ waveforms with a constant envelope.

Figure: Constant envelope.

The property-restoration approach can be described as follows. A signal may be designed with some invariant property, i.e. constant envelope of an angle modulated signal. If propagation through the channel or interference effects that degrade the receiver output also disturb this otherwise invariant property, then an algorithm can often be developed that senses this disturbance and adjusts the filter coefficients in such a way as to restore the invariant property. It can frequently be shown that restoring the property is tantamount to correcting the signal itself. When this is true, an adaptive filtering algorithm based on property restoration can be developed and specific knowledge of a reference signals is not required.

Figure 6.5

Finally, it should be noted that the resulting algorithms are very *signal-specific*. Obviously an algorithm to restore the constant envelope property of an FM signal will work very poorly on an AM signal.

The Constant-Modulus Adaptive Algorithm (CMA)

The constant modulus algorithm (CMA) was developed for blind channel equalization when the transmitted signal has a constant envelope such as digital modulation waveforms where the user's information is contained purely in the phasor angle, while the modulus or instantaneous amplitude is fixed to some value, *A*.

Assuming the input and output of the filter is complex-valued, we can define the error signal as

$$p(n) = |y(n)|^2 - A^2$$

and our cost function as the usual MSE

$$J = E[p^2(n)]\,.$$

As usual, we use a steepest-descent method to adjust the filter so as to minimize the cost function. Calculating the gradient of the cost function and employing instantaneous estimates we have for the filter update (CMA)

$$\boxed{\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) - \mu\left[|y(n)|^2 - A^2\right]y(n)\mathbf{u}^*(n)}.$$

Application of CMA to digital communications signals such as QPSK and 8PSK use a modified version of the adaptive equalizer known as the fractionally-spaced equalizer (FSE). The FSE approach helps to deal with associated pulse-shaping (used to control transmitted signals' power spectrum) which lead to non-constant modulus envelopes.