Kalman Filtering Experiment

Kalman Review and Problem Statement

Our linear dynamical system is described in state-space form

$$\mathbf{x}(n) = \mathbf{A}(\nu-1)\boldsymbol{\xi}(\nu-1) + \boldsymbol{\omega}(\nu)$$
$$\mathbf{y}(n) = \mathbf{X}(\nu)\boldsymbol{\xi}(\nu) + \boldsymbol{\varpi}(\nu)$$

Based on observations or measurements, $\mathbf{y}(n)$ we may optimally estimate the state $\mathbf{x}(n)$ with the Kalman filter as follows.

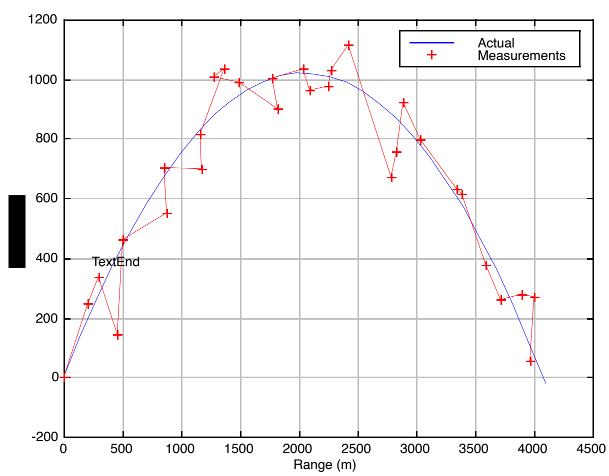
Initialization

$$\hat{\mathbf{x}}(0|0) = E[\boldsymbol{\xi}(0)]$$
$$\mathbf{P}(0|0) = E[\boldsymbol{\xi}(0)\boldsymbol{\xi}^H(0)]$$

Recursions

$$\hat{\mathbf{x}}(n|n-1) = \mathbf{A}(\nu-1)\hat{\boldsymbol{\xi}}(\nu-1|\nu-1)$$
$$\mathbf{P}(n|n-1) = \mathbf{A}(\nu-1)\boldsymbol{\Pi}(\nu-1|\nu-1)\mathbf{A}^H(\nu-1) + \boldsymbol{\Theta}_{\omega}(\nu)$$
$$\mathbf{K}(n) = \boldsymbol{\Pi}(\nu|\nu-1)\mathbf{X}^H(\nu)[\mathbf{X}(\nu)\boldsymbol{\Pi}(\nu|\nu-1)\mathbf{X}^H(\nu) + \boldsymbol{\Theta}_{\varpi}(\nu)]^{-1}$$
$$\hat{\mathbf{x}}(n|n) = \hat{\boldsymbol{\xi}}(\nu|\nu-1) + \mathbf{K}(\nu)[\boldsymbol{\psi}(\nu) - \mathbf{X}(\nu)\hat{\boldsymbol{\xi}}(\nu|\nu-1)]$$
$$\mathbf{P}(n|n) = [\mathbf{I} - \mathbf{K}(\nu)\mathbf{X}(\nu)]\boldsymbol{\Pi}(\nu|\nu-1)$$

As an example, a radar system provides range measurements (in both x- and y-directions) of a projectile and wish to predict range (trajectory), velocity, and acceleration (in both x- and y-directions) for the projectile.



System Model

We assume basic Newtonian laws of motion in the discrete model for the trajectory

$$d(n) = \delta(\nu-1) + \varpi(\nu-1)T + \frac{1}{2}\alpha(\nu-1)T^2$$
$$\varpi(\nu) = \varpi(\nu-1) + \alpha(\nu-1)T$$

where $d(n)$ is the distance or range, $v(n)$ is the velocity, $a(n)$ is the acceleration (all at time n), and T is the measurement sample period. We will use a three-state filter (one filter for the x- direction and one for the y-direction)

$$\mathbf{x}(n) = \begin{bmatrix} \text{Ρανγε}(\nu) \\ \text{ςελοχιτψ}(\nu) \\ \text{Αχχελερατιον}(\nu) \end{bmatrix}$$

and state equations given by

$$\mathbf{x}(n) = \mathbf{A}(\nu - 1)\boldsymbol{\xi}(\nu - 1) + \boldsymbol{\omega}(\nu)$$
$$\boldsymbol{\psi}(\nu) = \mathbf{X}(\nu)\boldsymbol{\xi}(\nu) + \boldsymbol{\varpi}(\nu)$$

where the state transition matrix (based on Newton's laws) is given by

$$\mathbf{A}(n) = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

and the measurement matrix (based on the fact we have only noisy range measurements) is given by

$$\mathbf{C}(n) = [1 \quad 0 \quad 0] \, .$$

Predictions and Results

We construct a three state Kalman filter for each direction (x- and y-). State initializations are given by

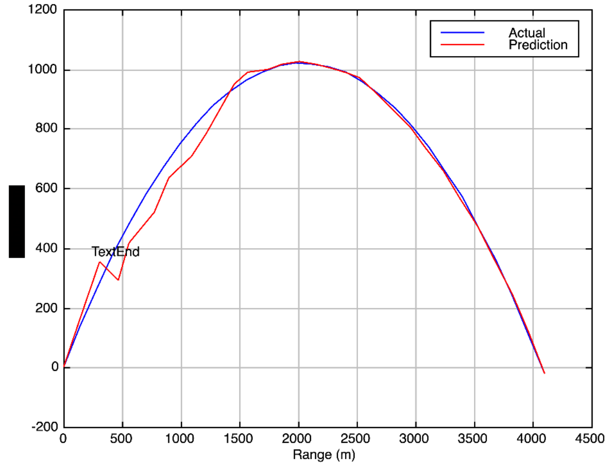
$$\mathbf{x}_x(0) = \begin{bmatrix} 0 \\ 141 \\ 0 \end{bmatrix}, \; \boldsymbol{\xi}_\psi(0) = \begin{bmatrix} 0 \\ 141 \\ -9.8 \end{bmatrix}$$

where we assume the projectile has an initial velocity of 200 m/s at an angle of π / 4 radians from horizon. Error covariance initializations are given by

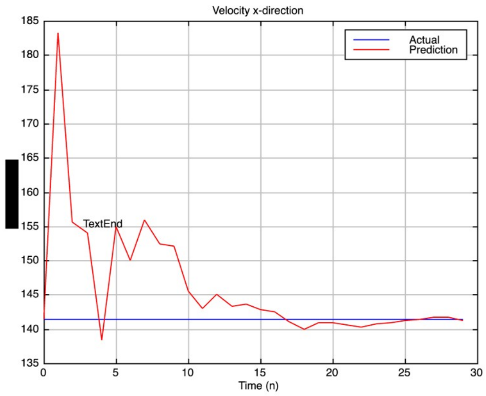
$$\mathbf{P}_x(0|0) = \boldsymbol{\xi}_\xi(0)\boldsymbol{\xi}_\xi^H(0), \; \boldsymbol{\Pi}_\psi(0|0) = \boldsymbol{\xi}_\psi(0)\boldsymbol{\xi}_\psi^H(0) \, .$$

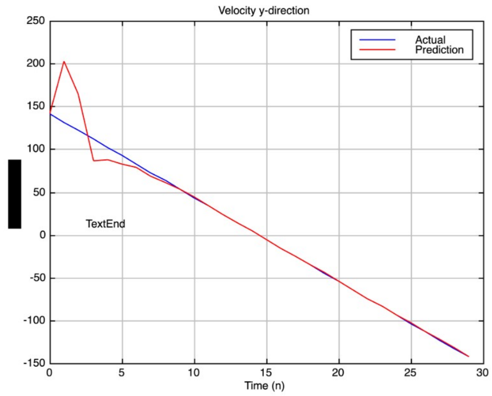
These initializations can be changed to whatever a user desires. Finally the sample period is given by $T = 1$; process noise variance $\sigma_w^2 = 0$ (neglect friction, air drag, etc...); and measurement noise $\sigma_v^2 = 10^4$.

Range (x- and y-directions)

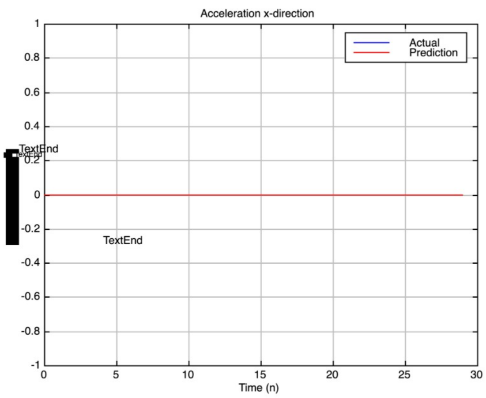


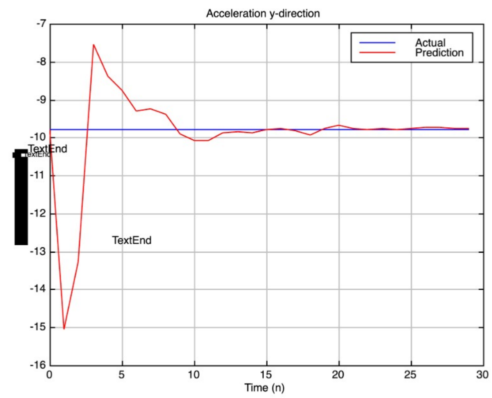
Velocity



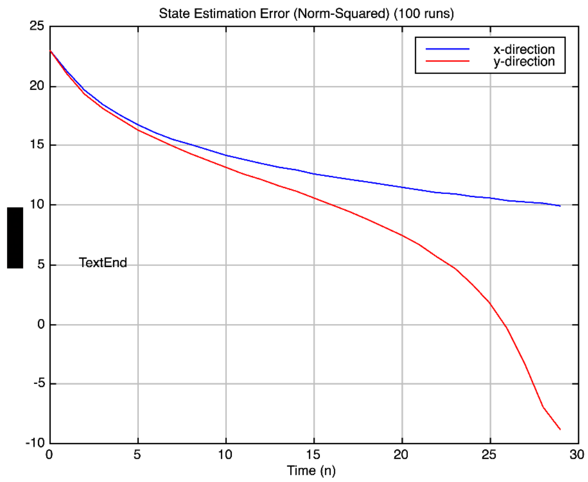


Acceleration





The state estimation error (norm squared), averaged out over 100 runs is given as



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%-----------------------------------------------------------------------------
% KALMAN DEMO
%
% This MATLAB code utilizes the Kalman filter to predict state information
% regarding the launching of a projectile.  The filter is only given range
% measurements (in x and y directions) perhaps from someone with a telescope.
% State information includes range, velocity, and acceleration (in both x
% and y directions) of the projectile.
%-----------------------------------------------------------------------------
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(MATLAB code for this simulation can be found on the EE594 web page under related links).

Some Applications of the kalman Filter

The Kalman filter can be used for a variety of end purposes. Its basic function is to provide estimates of the current state of the system. But it also serves as the basis for predicting future values of prescribed variables or for improving estimates of variables at earlier times.

J. Campell, S. Synnott, and G. Bierman, “Voyager Orbit Determination at Jupiter,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 256-268, Mar. 1983.

R. Berg, “Estimation and Prediction for maneuvering target Trajectories,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 294-304, Mar. 1983.

> Comments: The classical problem of anti-aircraft gun fire control is the accurate prediction of the future position of a given target at the time of projectile intercept. Having obtained this information, correct gun-pointing angles can be ascertained.

M. Kao and D. Eller, “Multiconfiguration kalman Filter Design for High-Performance GPS Navigation,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 304-314, Mar. 1983.

P. Fung and M. Grimble, “Dynamic Ship Positioning Using a Self-Tuning Kalman Filter,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 339=349, Mar. 1983.

> Comments: A dynamic positioning system is used to maintain a floating vessel on a specified position and at a desired heading.

M. Sidar and B. Doolin, “On the Feasibility of Real-Time Prediction of Aircraft Carrier Motion at Sea,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 350-355, Mar. 1983.

Comments: The landing phase of an aircraft aboard an aircraft carrier represents a complex operation and a demanding task. The last 10-15s before aircraft touchdown are critical and involve terminal guidance and difficult control problems because not only is the aircraft disturbed by several kinds of stochastic wind disturbances but also the touchdown point on the ship is being moved randomly. Despite this, the landing accuracy specified for carrier operations is very high.

V. Lumelsky, “Estimation and Prediction of Unmeasureable Variables in the Steel Mill Soaking Pit Control System,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 388-400, Mar. 1983.

Comments: The objective of the soaking pit operation in the steel-making process is to equalize the temperature throughout the steel ingot masses at some prespecified temperature before the ingots may be rolled at the rolling mill. The ingot temperature, though, is not measurable directly and must be estimated using some indirect measurements (pit wall temperature, fuel flow, etc.). Another piece of information which is important for the whole operation is the predicted moment of time at which the ingots will arrive at the said temperature.

J. Tylee, “On-Line Failure Detection in Nuclear Power Plant Instrumentation,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 406-415, Mar. 1983.

B. Leibundgut, A. Rault, and F. Gendreau, “Application of Kalman Filtering to Demographic Models,” *IEEE Trans. Automatic Control*, vol. AC-28, pp. 427-434, Mar. 1983.

Comments: The problem is estimating the number of male and females in the French cattle herd per age class using available data from different statistical services. From these simulation models, various agricultural econmic policies may be simulated.