Introduction

EE594 should be titled "Adaptive and Optimal Signal Processing"

Preliminary

Definition *Filter*–A device that is applied to a set of (noisy) data in order to extract information about a prescribed quantity of interest.

Example: Lowpass filtering extracts low frequency signal components.

A filter is used to perform three basic information or signal processing tasks:

1) **Filtering** extraction of information about a quantity of interest at sample n using data measured up to and including sample n, i.e.

$$y(n) = \frac{1}{2}y(n-1) + x(n) - x(n-1).$$

2) **Smoothing** extraction of information about a quantity of interest at sample n using data measured up to and after sample n, i.e.

$$y(n) = x(n+1) + x(n) + x(n-1).$$

There is an inherent delay in producing the quantity of interest.

3) **Prediction** extraction of information about a quantity of interest at sample n+1 using data measured up to (but not including) sample n, i.e.

$$\hat{x}(n+1) = \alpha\hat{x}(n) + (1-\alpha)x(n).$$

Optimal Filters

Op•ti•mal *adj*. Most favorable, best

In this course we will design *optimal filters*–filters which perform a task or function better than any other filter according to some objective criteria.

Example: A matched filter (MF) is the "best" filter in terms of maximizing the output signal-to-noise ratio (SNR) of an input signal composed of a known deterministic signal in noise. The solution is elementary in communication theory:

$$h(t) = g(T-t)$$

where $h(t)$ is the impulse response of the MF, $g(t)$ is the pulse signal to match against, and T is the pulse duration.

Figure: Matched filter

Example: An eigenfilter is the "best" filter in terms of maximizing the output signal-to-noise ratio (SNR) of an input signal composed of a random signal in noise. We assume knowledge of the statistics of the random signal (i.e. correlation matrix). This is the stochastic version of the MF.

Figure: Eigenfilter

Example: A Wiener filter, **w** is designed to process a given input signal, $x(n)$ and form the "best" estimate, $y(n)$ of a related signal, $d(n)$ called the desired signal.

Figure 5.1: Wiener filter

Since $x(n)$ and $d(n)$ are not known in advance and can only be described statistically, the solution is NOT

$$W(e^{j\omega}) = D(e^{j\omega}) / X(e^{j\omega}) .$$

In this case, "best" is in a mean-squared error (MSE) sense

$$J = E[|e(n)|^2] .$$

Example: A Kalman filter recursively computes the "best" estimate for *non-stationary* (statistics are time-varying) processes.

Adaptive Filters

A•dap•tive *adj*. Tending or able to adapt

A•dapt *v.t.* To put in harmony with changed circumstances || to make more suitable by altering

In this course we will also design *adaptive filters* or sometimes called *supervised adaptive filters*–filters which make themselves more suitable to performing a task or function by altering themselves. These filters are used when statistics are unknown but a reference or desired or training signal is available. Here the desired response is compared to the actual response of the filter, and the resulting error is used to adjust the filter coefficients.

Figure:

The three basic components

1) initial estimate
2) feedback (measures quality of estimate)
3) algorithm to refine estimate based on feedback

Example.

Adaptive signal processing in essence is developing algorithms to adjust or adapt filter coefficients to filter noisy signals. There is no one solution to this problem and the choice of algorithms is determined among other things by:

1) *Performance*. If the input to the adaptive filter is stationary and not time-varying, how fast does the adaptive filter converge "close-enough" to the Wiener filter? How close is "close enough"?

2) *Robustness*. Small disturbances should result in small estimation errors.

3) *Computational requirements*. How many MACs are required to adjust the filter per input sample?

The challenge is to 1) understand the capabilities and limitations of the various algorithms and 2) use this understanding in the selection of the appropriate algorithm.

Finally, in this course we will also (attempt) to design *blind adaptive filters* or *unsupervised adaptive filters*–filters which are used when statistics are unknown and a reference is unavailable. The design of algorithms for blind adaptation is one of the most popular research areas today and among the most challenging.

Example. Mobile telephone and blind equalization

Applications of Adaptive Filters

One basic common feature of all applications is an input signal $u(n)$, desired (or reference) signal $d(n)$ used to compute an estimation error $e(n)$, which in turn is used (feedback) to control the adjustment of the filter coefficients (next guess). The output signal of the adaptive filter is denoted $y(n)$.

There are four classes of adaptive filtering applications (or configuration).

1) Identification (or system identification or system modeling)

Figure 7a

Examples: system identification (acoustic echo cancellation)

2) Inverse modeling

Figure 7b

Examples: adaptive equalization (p.34)

3) Prediction

Figure 7c:

Examples: linear predictive coding (LPC) (p. 39)

4) Interference Canceling

Figure 7d

Examples: adaptive noise canceling (p. 50), (line) echo cancellation

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History

(1795) Gauss (age 18) invented method of least squares and starts linear estimation theory

(1940s) Kolmogorov develops linear prediction for discrete-time stationary processes

(1940s) Wiener develops linear prediction for continuous-time stationary processes and the filtering problem of estimating a process corrupted by additive noise (Wiener-Hopf integral equation).

(1947) Levinson formulates Wiener filtering problem for discrete-time and develops elegant algorithm to solve it.

(1959) Widrow and Hopf develop LMS algorithm

(1960) Kalman develops procedure for estimation of linear, dynamical systems

(1994) Sayed and Kailath relate RLS algorithm and Kalman filter

First "big" deployment of adaptive filtering is probably due to Lucky for adaptive equalization of the telephone channel (1965)