Homework #4 LMS Adaptive Filters
(due Friday, Sep. 27, 2002)

Build the `LMS.M` tool in the Adaptive Signal Processing toolkit.

Notes on this and future homeworks:

1) For experiments involving Monte-Carlo-style simulations, make sure the random number generator seed is reset so that comparisons have the same random signal data. See `randn('state',0)`.
2) You can compute the actual J_{ex} by averaging the last 100 or so MSE values (assuming steady-state) from your experiment and using the definition $J(\infty) = J_{min} + J_{ex}(\infty)$.
3) Readability of the learning curves can be increased by MA filtering the MSE data and/or downsampling the data (this is a common technique). Be sure to indicate length of MA filter (if applied). Note that the `MSE_plot.m` already has a downsampliong option.

Experiments

1) (Step-size experiments) Redo Experiment 1 Case 1 in Section 8.4 of your text using the LMS algorithm to update the adaptive filter. Note that for the one-step predictor (length 2 FIR) in Figure 8.7, $\mathbf{u}(n)$ is the desired signal and $\mathbf{u}(n-1)$ is the input signal. Also allow your AR process some settling time (t_s ~ 20 samples) before feeding it to the adaptive filter.

(a) Verify that choosing $\mu = 0.3$ is within the upper bound for convergence in the mean square but does not provide convergence of the error signal. You might get lucky and the error will converge but a few trials will get the point across. Comment.

(b) Plot trajectories of $\mathbf{w}(n)$ with $\mu = 0.01$, 0.001, and 0.0001. Compare to Figure 8.9 (use contour values from SD on Homework #3). Give the $\mathbf{w}(L)$ for each μ and compare with the Wiener filter. Comment on noisy trajectory, "wandering at the bottom of the error surface," and other properties we discussed. Take L ~ 50000 so that the MSE is in steady state for the small μ. Note that even with these step-sizes, the misadjustment will still be large.

(c) On a single figure plot the MSEs (learning curves) in dB (averaged over say 25 or 100 runs) for the step-sizes in (b). A sample code segment might be:

```
L = 50000;
ts = 20;
randn('seed',0);  % reset random number generator seed
for runs = 1:100
   u = AR_synthesizer(a,L);
   [e,w] = lms(w_init,u(ts:L-1),u(ts+1:L),mu)
   accumulated_squared_e = accumulated_squared_e + e.^2;
end;
MSE = accumulated_squared_e / 100;
MSE_plot(MSE);
```

Do the same plot but with downsampled ($D = 100$) MSE data. Do the same plot but with MA-filtered MSE data (length 100 MA filter). Do the same plot but with downsampled and MA-filtered MSE data. Comment on the convergence rates.

(d) On separate figures, plot the downsampled and MA-filtered MSEs from (c) in normal units and indicate J_{min} from Table 8.1, actual J_{ex}, and the level of misadjustment on your figure. How does this compare with the theoretical value given in (9.90) and corrected in Lecture Notes? In order to accurately determine J_{ex} from your experiment, you should average the last 100 MSEs at convergence.

2) (Eigenvalue spread experiments) Redo Experiment 1 (all cases) in Section 8.4 of your text using the LMS algorithm to update the coefficient vector.

(a) Plot trajectories of $\mathbf{w}(n)$ for each case and compare to Figure 8.9 (use contour values from SD on Homework #2). For this part, take L ~ 25000 so that the MSE is in steady state for large χ and $\mu = 0.01$. Give the $\mathbf{w}(L)$ for each case and compare with the Wiener filter. Comment.

(b) On separate figures, plot the MSEs for cases 1 and 3 in normal units. Choose appropriate step-sizes so that J_{ex} is roughly equal in both cases (this will require some trial and error and possible non-zero filter initialization). Indicate J_{min} from Table 8.1, actual J_{ex}, and the level of misadjustment on your figure. How does this compare with the theoretical value given in (9.90) and corrected in Lecture Notes? Comment on the convergence rates for each case.

3) (Adaptive Prediction Experiment) Code the computer experiment on adaptive equalization in Section 9.6 of your text. Reproduce (to within the limits of Monte-Carlo-style simulations) Figures 9.14 and 9.16. Use a dB scale and don't worry about MSE normalization [artificially setting $J(0) = 1$] for Figure 9.16.

Text

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