Homework #2 Eigenanalysis
(due Sept. 13, 2002)

For the first four problems, consider the filtering problem illustrated in Figure 4.4.

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1. (Noisy sinusoids)

(a) Given a signal $u(n)$ and a noise signal $v(n)$ (both zero mean) we wish to determine the variance of the noise signal, σ_v^2 such that the signal-to-noise ratio (SNR) of the mixture, $x(n) = u(n) + v(n)$ has some desired value in dB. Solve for σ_v^2:

$$SNR_{dB} = 10\log_{10}\frac{\sigma_u^2}{\sigma_v^2}.$$

(b) Let $u(n) = \sin(\omega_0 n)$ with $\omega_0 = \pi / 4$; $v(n)$ be white, Gaussian noise; and $x(n)$ have a 3dB SNR. Plot 128 samples (total length of signals) of $u(n)$, $v(n)$, and $x(n)$ and the 1024-point PSD, $S_x(\omega)$ (use your periodogram tool). You can generate $v(n)$ with the MATLAB command

```
v = randn(128,1) .* sqrt(sigma_v2); % sigma_v2 = desired variance
```

2. (Eigenfilter for a Noisy Sinusoid) Review Section 2.4.

(a) For a complex sinusoid ($\omega_0 = \pi / 4$) plus noise, determine the FIR filter **w** (length 32) which maximizes the SNR. You can use MATLAB's `'toeplitz'` and `'eig'` functions. Plot the filter's coefficients (real and imaginary parts), magnitude response, and pole/zero pattern. Why does your eigenfilter "make sense?"

(b) Apply eigenfilter from (a) to the noisy sinusoid generated in Prob. 1(b). Plot the PSD of the filter output, $S_y(\omega)$. Comment on the difference between $S_x(\omega)$ [from Prob. 1(b)] and $S_y(\omega)$.

(c) How would your filter's magnitude response change if the length was greater?

3. (Eigenfilter for Noisy Highpass AR Processes) Let $u(n)$ be an AR process generated by filtering zero-mean, unit-variance, white, Gaussian noise through the filter with AR coefficients

$$\mathbf{a} = [1 \quad 0.5]^T.$$

(a) Plot the magnitude response of the filter.

(b) Let $u(n)$ be length 16,000; $v(n)$ be a zero-mean white Gaussian noise; and $x(n) = u(n) + v(n)$ have a 0dB SNR.. Plot the PSD of $u(n)$ and $x(n)$ (noisy AR process).

(c) Plot the impulse response, magnitude response, and pole/zero diagram of the eigenfilter (length 16) designed to maximize the SNR. Comment on why your eigenfilter is a reasonable design given the PSD in (b).

(d) Plot the PSD of output of the eigenfilter, $y(n)$.

4. (Eigenfilter for Noisy Speech) Let $u(n)$ be your speech signal of the utterance "0-1-2-3-4-5-6-7-8-9," $v(n)$ be a zero-mean white Gaussian noise, and $x(n) = u(n) + v(n)$ have a 3dB SNR.

(a) Plot the PSD of $u(n)$ and $x(n)$ (noisy speech). Listen to $x(n)$—you may have to normalize the sample values to prevent playback distortion on the sound card:

```
x(n) = x(n) ./ max(abs(x(n)));
```

(b) Plot the impulse response and magnitude response of the eigenfilter (length 16) designed to maximize the SNR. Comment on why your eigenfilter is a reasonable design given your speech PSD.

(c) Plot the periodogram of output of the eigenfilter, $y(n)$. Listen to $y(n)$—does the SNR appear to have improved? Try filtering the noisy speech signal with other filters (random coefficient vectors make good "other" filters)–does the output sound better in an SNR sense?

5. (Wiener filter)
(a) Create a desired signal, $d(n)$ by filtering your input speech signal, $u(n)$ through the filter

$$\mathbf{h} = [1 \quad 0.5 \quad 0.4 \quad 0.3]^T .$$

Compute the 4×4 correlation matrix, $\mathbf{R}$ of the speech signal, and the 4×1 cross correlation vector, $\mathbf{p}$ between the speech signal and the desired signals.

(b) The Wiener filter, $\mathbf{w}_{\mathrm{opt}} = \mathbf{R}^{-1}\mathbf{p}$ is designed to minimize the squared error between the desired signal and the Wiener filter output. Intuitively we could achieve minimum squared error if $\mathbf{w}$ is "close" to $\mathbf{h}$. Compute and list the Wiener filter and compare to $\mathbf{h}$. Note that the Wiener filter is computed only from $u(n)$ and $d(n)$–nothing about $\mathbf{h}$ is known.

(c) Repeat (b) for a length 3 Wiener filter. Comment.

(d) Repeat (b) for a length 5 Wiener filter. Comment.

Text

Chapter 4 #5, 10, 13, 15
Chapter 5 #2a, b;