EE 594 - Adaptive Signal Processing

Fall 2002 – Exam #1

This is a take-home examination due at the beginning of class on Wednesday, October 9, 2002.

"The attached solution is due entirely to my own, individual efforts. In no way have I consulted with anyone other than (possibly) the instructor of this course in creating these solutions."

Signature: ____ Date: _________

Print: ____

Part 1: Take Home	
Problem 1	/ 20 points
Problem 2	/ 20 points
Problem 3	/ 20 points
Problem 4	/ 25 points
Part 2: In-class	
Problem 1	/ 8 points
Problem 2	/ 7 points
Total	/ 100 points

Problem 1

Consider the equalizer in Fig. P1. Here, $s(n)$, $v(n)$ are real-valued, white, zero-mean, Gaussian noise processes with $\sigma_s^2 = 10$, $\sigma_v^2 = 1$, respectively. Also $s(n)$ and $v(n)$ are drawn from independent noise sources; and

$$H_1(z) = \frac{1}{1 - 0.5z^{-1}}, \; H_2(z) = \frac{1}{1 + 0.8z^{-1}}.$$

You will calculate the length-3 Wiener filter, $\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{p}$.

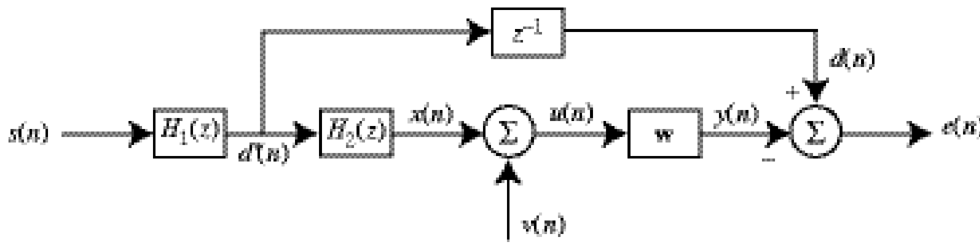

Figure P1: Equalizer

(a) You can write

$$\mathbf{R} = \mathbf{R}_x + \mathbf{R}_v$$

where $\mathbf{R} = E[\mathbf{u}(n)\mathbf{u}^H(n)]$, $\mathbf{R}_x = E[\mathbf{x}(n)\mathbf{x}^H(n)]$, and $\mathbf{R}_v = E[\mathbf{v}(n)\mathbf{v}^H(n)]$. Why?

(b) Determine the 3 × 3 matrix, $\mathbf{R}$.

(c) Determine the 3 × 1 vector, $\mathbf{p}$.

(d) Compute $\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{p}$

(e) (BONUS +10) Verify your answer (to within statistical limits) by implementing the equalizer using an LMS adaptive filter with a small μ. This way you'll know (indirectly) whether your work in (b) and (c) is correct.

Problem 2

In this problem we apply the steepest-descent algorithm to the adjustment of a length-2 filter. Let

$$\mathbf{R} = \begin{bmatrix} 1 & 0.4045 \\ 0.4045 & 1 \end{bmatrix}, \mathbf{p} = \begin{bmatrix} 0 \\ 0.2939 \end{bmatrix}, \sigma_d^2 = 0.5.$$

(a) Determine the theoretical MSE at time *n*.

(b) Determine (using MATLAB) the eigenvalues and eigenvectors or **R**.

(c) With $\mu = 0.1 / \lambda_{max}$ and $\mathbf{w}(0) = [-1 \quad -2]^T$, plot the MSE from your steepest-descent tool.

(d) With $\mu = 0.1 / \lambda_{max}$ and $\mathbf{w}(0) = [-1 \quad -2]^T$, plot the trajectory of the filter coefficients. On your plot, also draw the eigenvectors and label each vector as λ_{min} or λ_{max} depending on which eigenvalue it is associated with.

(e) Identify the two modes (fast and slow) in the trajectory plot and discuss how each is related to λ_{min} or λ_{max}.

Problem 3

All hands-free telephones or speakerphones generate an acoustic echo for the user on the remote side. The phenomenon is illustrated in Fig. P2 below. In the setup, the remote user's speech, $u(n)$ is transmitted down the telephone line to the local speakerphone. The speech is reproduced at the loudspeaker, echoed throughout the room, and picked up by the microphone. The return signal (ignore y for now), $d(n) = \mathbf{h} * \mathbf{u}(n)$ is an echoed-version of remote user's speech and is heard by the remote user on their telephone. Here $\mathbf{h}$ is the impulse response of the room.

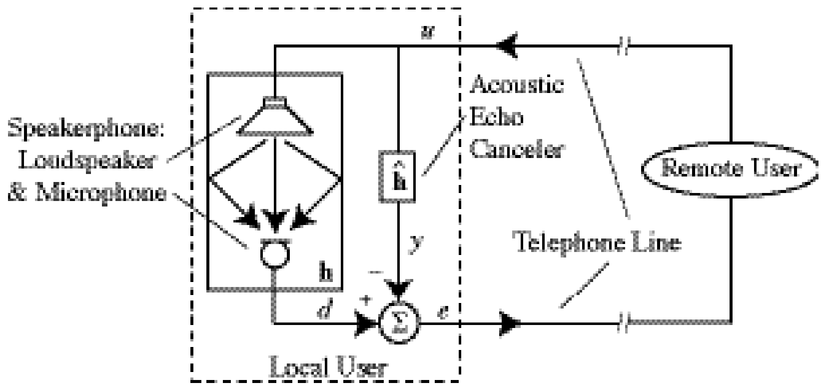

Figure P2: Acoustic echo cancellation setup.

Modern, full-duplex speakerphones include an LMS adaptive filter, $\hat{\mathbf{h}}(n)$ (arranged in a system-modeling configuration) which is used to synthesize an anti-echo, $-y(n)$ as in Fig. P2. When added to $d(n)$, the resulting return signal, $e(n)$ is (hopefully) zero, i.e. we have created a perfect anti-echo.

Download the following MATLAB data file (zipped for convenience)

`http://www.ece.nmsu.edu/~pdeleon/Teaching/EE594/Exam1.zip`

which contains u and d. Assume everything in this problem has a sample rate, f_s = 8000Hz.

(a) Listen to u and d. Assuming a reverberation time of 0.25 seconds, what is the minimum length, M that you select for your adaptive filter?

(b) Construct an acoustic echo canceller and listen to the return signal, $e(n)$. Comment.

(c) Plot the MSE of your acoustic echo canceler for the one run in (b). Be sure to MA filter and downsample for readability.

(d) Plot the final $\hat{\mathbf{h}}$ which is returned to you by your LMS tool. This is your estimate of the room impulse response, $\mathbf{h}$.

(e) If you listen carefully to d and think about what you have for the final $\hat{\mathbf{h}}$, it makes good sense audibly. Why?

Problem 4

Redo the computer experiment on adaptive equalization in Section 9.7 (both experiment 1 and 2) in your text. Note that the correct condition for convergence in the mean square is

$$0 < \mu < \frac{2}{3\,\mathrm{tr}(\mathbf{R})} = \frac{2}{3M\sigma_u^2} = \frac{2}{3(\text{tap input power})}.$$

Also note the following typos on p. 416:

$$u(n) = \sum_{k=1}^{3} h_k x(n-k) + v(n)$$

(*a* should be *x*) and

$$r(0) = h_1^2 + h_2^2 + h_3^2 + \sigma_v^2$$

(h_3 is squared not cubed).

In the experiment assume, *x* is uniformly distributed (rand) whereas *v* is white, Gaussian noise with variance σ_v^2 = 0.001 (randn). In MATLAB we would have

```
x = sign(rand(L,1)-0.5);
v = sqrt(0.001)*randn(L,1)
```

(a) Determine the correct value of μ_{crit}, i.e. upper bound of the step-size using the lecture notes. In order to stay consistent with the experiment and plots, please use the same values for μ in your codes as published in the text.

(b) Recreate Fig. 9.21 but in units of dB (don't worry about the circle, triangle, square, diamond—legends in ink are fine).

(c) Recreate Fig. 9.23 but in units of dB.

(d) Haykin uses a linear-phase/phase delay argument for justifying a delay, δ = 7. Assuming an experimental setup like the one used in (b) but with only *W* = 3.1, fill in the table below. Consider averaging the last 100 samples of the MSE (in normal units) to estimate the steady-state MSE in normal units. Use at least 6 digits of precision.

δ	Steady-State MSE	δ	Steady-State MSE
0		5	
1		6	
2		7	
3		8	
4		9	

Explain whether you are comfortable or not with δ = 7?

Problem 1

In Fig. P3, are four MSE plots that correspond to different variations of the adaptive equalizer from Section 9.7. The “base” experiment is indicated by plot #3. Where appropriate, misadjustment has been calibrated.

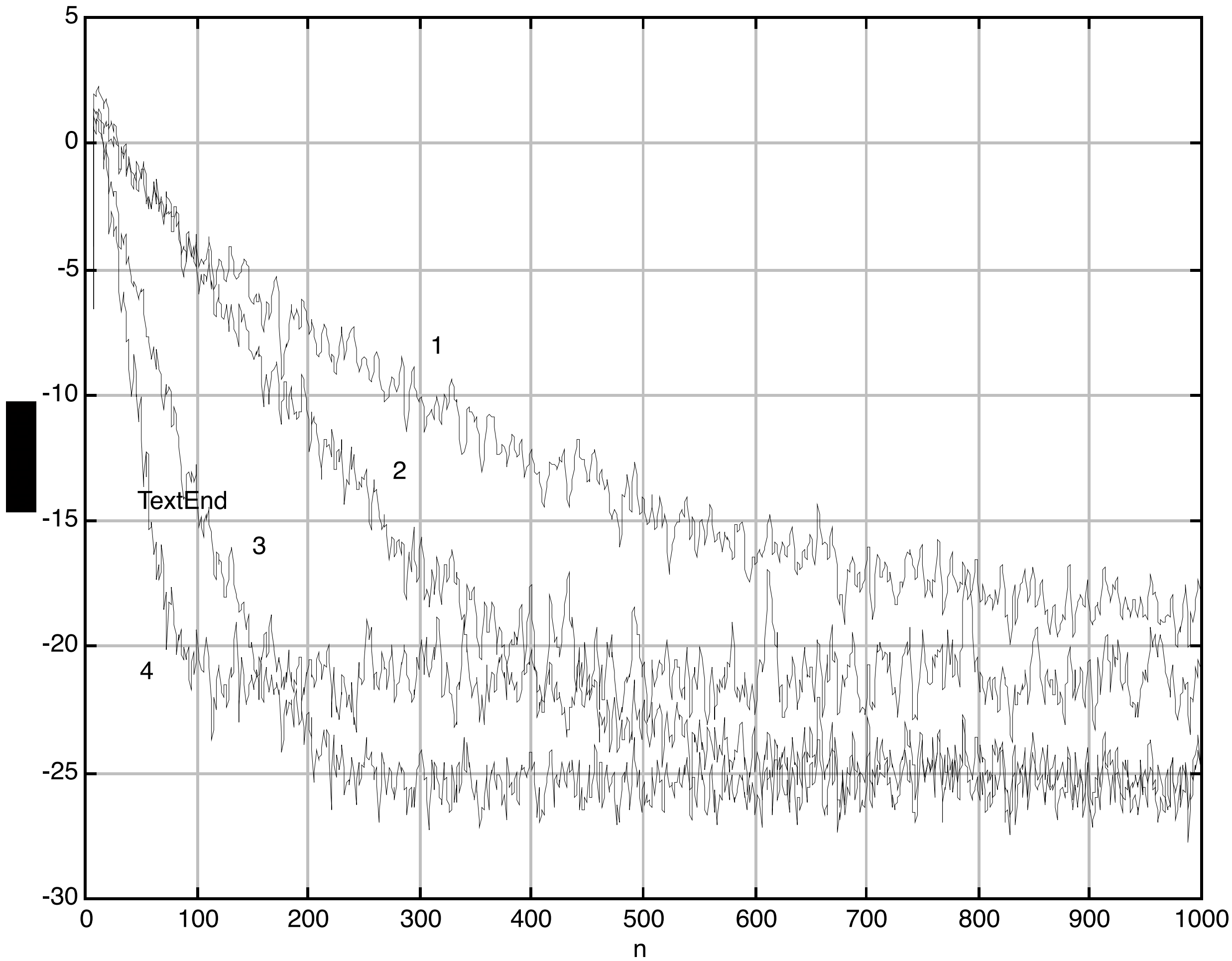

Figure P3: MSE of “base” experiment plot #3, plus MSE plots from variations of the base experiment.

Indicate which plot number corresponds to the variation described **and** provide comment(s) justifying your answer.

“Base” Experiment. Plot # __3__

(a) Eigenvalue spread is larger than in “base” experiment. Plot # _____. Comment.

(b) Step-size is larger than in “base” experiment. Plot # _____. Comment.

(c) Filter has more coefficients than in “base” experiment. Plot # _____. Comment.

Problem 2

(a) Let $\mathbf{m}(n)$ denote the *mean weight vector* in the LMS algorithm at iteration n; that is

$$\mathbf{m}(n) = E[\hat{\mathbf{w}}(n)] \,.$$

Using the independence assumption of Section 9.4, show that

$$\mathbf{m}(n) = (\mathbf{I} - \mu\mathbf{R})^n [\mathbf{m}(0) - \mathbf{m}(\infty)] + \mathbf{m}(\infty)$$

where μ is the step-size parameter, $\mathbf{R}$ is the correlation matrix of the input vector, and $\mathbf{m}(0)$ and $\mathbf{m}(\infty)$ are the initial and final values of the mean weight vector, respectively. You may assume $\mathbf{m}(\infty) = \mathbf{w}_{opt}$.

(b) Hence, show that for convergence of the mean value, $\mathbf{m}(n)$, the step-size parameter μ must satisfy the condition

$$0 < \mu < \frac{2}{\lambda_{\max}}$$

where $\lambda_{\max}$ is the largest eigenvalue of the correlation matrix $\mathbf{R}$.

Note:

"Convergence of the Mean" is equivalent to $\lim_{n \to \infty} E[\varepsilon(n)] = \mathbf{0}$

"Convergence of the Mean Value" is equivalent to $\lim_{n \to \infty} E[\hat{\mathbf{w}}(n)] = \mathbf{w}_{opt}$

These, however, imply the same result.